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Modeling of Micro-Hydropower Systems: The Case of Hhaynu Micro-Hydro Power Plant in Tanzania

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Abstract: ***Mini and Micro-hydropower plants are used to supply electricity to the rural and off-grid areas of many developing countries like Tanzania. Their power capacity ranges from lower capacity of 5 kW to a higher capacity of 100 kW which is equivalent of supplying electricity from a few households to several villages in the region. The main challenge that have influenced to undertake this research study is centred on the possibility of modelling and performance optimization of a designed micro-hydro turbine system that can meet the dynamic load demand from the growing rural and off-grid users of many developing countries like Tanzania and also at the same time achieve high energy utilization efficiency with minimum energy losses. The methods used in this research study are based on the field work and site data measurements that lead to determination of hydraulic and turbine transfer functions as inputs to system design and modelling that will optimise the system performance characteristics. The results from data analysis and modelling shows that the feasible water flow discharge for the designed micro-hydropower plant is 0.45 m^3/s with the gross head of 25m that gives a turbine output (mechanical)power of 79.5 kW.***

Keywords: Micro-hydro turbine, electricity, penstock pipe, transfer function, Hhaynu, Tanzania

1. Introduction

Micro-hydropower plant modelling can be done by taking into consideration the physical arrangement of its hydraulic turbine system. The general arrangement of the micro hydro turbine system consists of a weir, intake, canal, forebay, penstock and turbine/generator system. The weir is used to divert the water from a river and direct it to the intake and canal. At the end of the canal, there is a forebay which is a temporary water storage tank and is usually connected to an elevated head penstock (pipe) that carry water flow discharge with high pressure and velocity to the water turbine unit which is connected to the generator that rotated and produces electricity as shown on Figure 1.1 below.

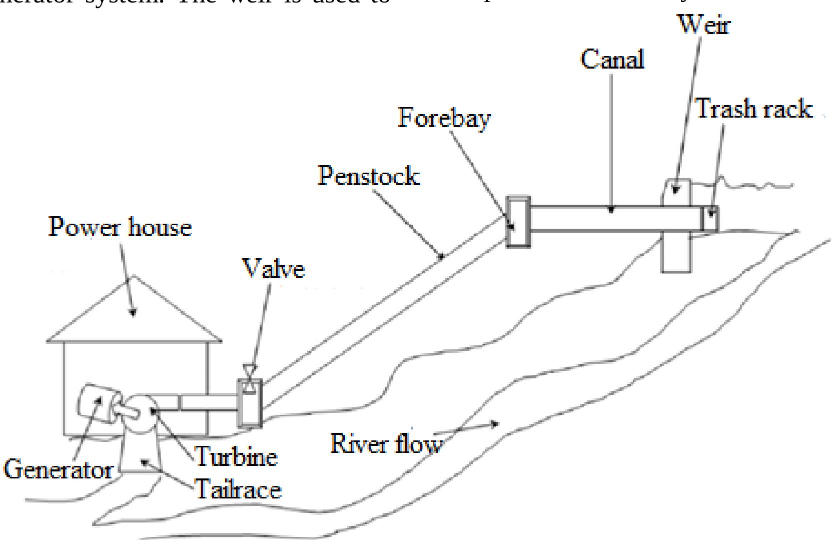

Figure 1.1: Schematic diagram of a micro-hydropower plant [1]

In micro-hydro-turbine modelling, the tangential characteristics of the micro-hydropower plant are determined by the penstock dynamics, turbine dynamics, turbine control dynamics, generator dynamics and load dynamics as shown in Figure 1.2.

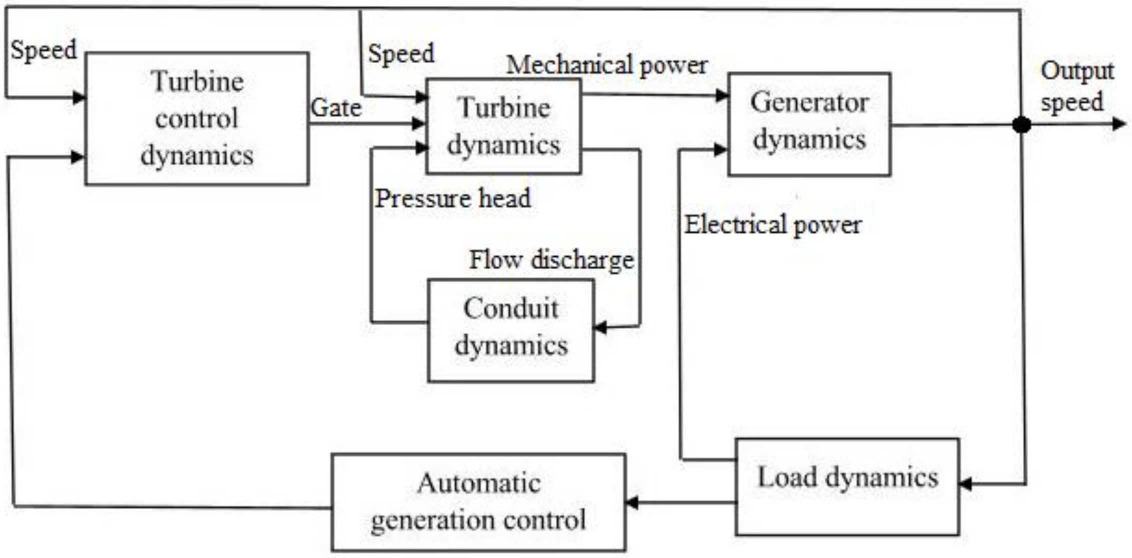

Figure 1.2: Hydropower plants functional block modelling diagrams

The turbine conduit dynamics is related to the dynamics of the water flow in the penstock which is determined by the water flow rate value *Q*, pressure head *H*, flow velocity *U* and the length of the penstock *L*. In this relationship, the turbine and penstock dynamics are determined by the four basic relations between the turbine power P_m, velocity of the water in the penstock *U* and the acceleration of water column *H* as shown inFigure1.3 below.

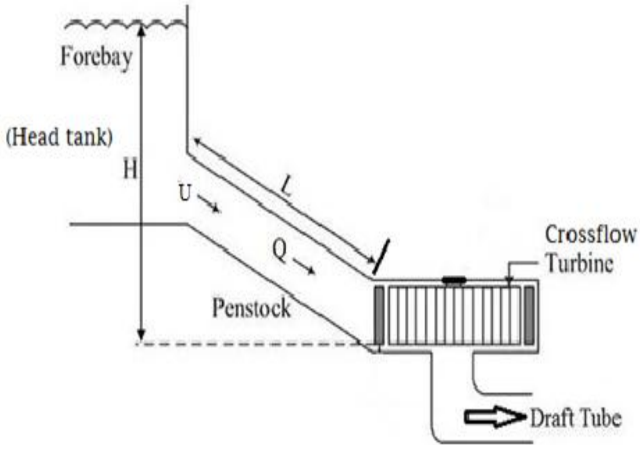

Figure 1.3: Components layout of a hydro turbine system [2]

On the other hand, the turbine/generator dynamics are determined by the power/torque and the rotational speed of the prime mover which has to be maintained and controlled by turbine control dynamics through gate position from the servo motor. With linear blocks, the micro hydropower plant model can be presented as shown in Figure 1.4 below as a closed system with a feedback loop. The plant dynamics consists of gate positioner, hydraulic system and turbine/generator system. On the other hand, load disturbances are related to the unpredicted load that may be introduced into the system and may affect the system performance and stability by reducing/increasing the generator rotational speed and hence the frequency.

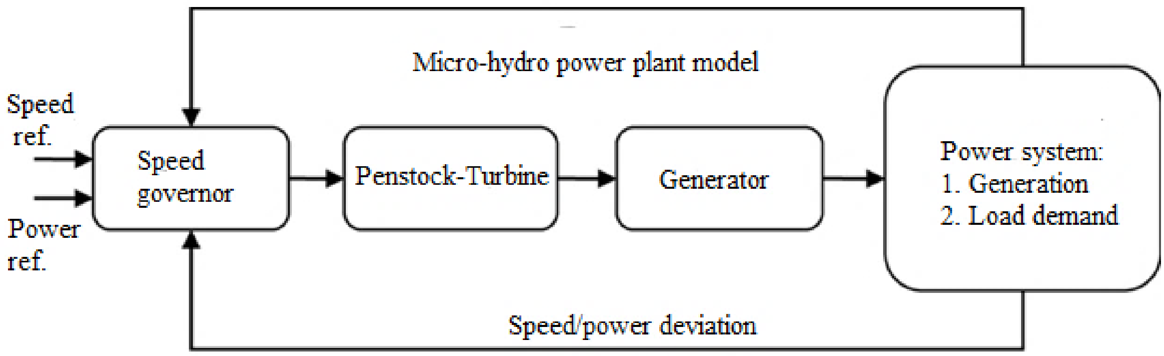

Figure 1.4: Micro-hydro power plant block diagram

2. Methods and Materials

2.1 Hydraulic Turbine dynamics

Electricity generated from micro hydropower plants is an important renewable energy source in many rural areas of the developing countries which provide significant flexibility during its operation phase (off-grid operation). When most of the micro hydro turbine is in operation, their dynamic operating behaviour is mostly determined by the transient characteristics of the water flow in the penstock, although from the literature it is noted that the turbine energy conversion process has been involved non-dynamic characteristics [3]. In broad categories, micro-hydro power plants system models can be classified into two major segments based on the complexity of their transfer functions/equations involved in the modelling process i.e. the linear models which sometimes called non-elastic model and non-linear models which sometimes called elastic model.

In the modelling of a micro-hydro turbine system, the use of transfer functions/equations with linear and non-linear behaviour has to be used. The transfer functions are mathematical equations which are essential tools for the simulation of micro hydro turbine systems which will be explained in details in the following sub-sections.

2.2 Hydraulic Turbine transfer function

Most linear hydropower models are calculated around an operating point and they are extracted from basic equations of turbine and penstock hydraulic characteristics. The model equations are based on the transfer function which relates to the turbine mechanical output power to the gate opening signal. Considering the studies based on system stability, the formulation of the representation of the hydraulic turbine is based on the following assumptions:

(a) The penstock pipe is in-elastic and water is incompressible
(b) The hydraulic resistance in the system is negligible
(c) The water flow velocity in the pipe varies directly with the gate opening and with the root square of the net head
(d) The hydraulic turbine output power is proportional to the product of the head and volume of flow

2.3 Penstock Pipe and Turbine system

In a micro hydro turbine system, the penstock is the pipe which carries water discharge from the head tank (forebay) to the turbine unit. The velocity of the water in the penstock will be determined by the pressure head/elevation between the intake and the powerhouse. The appropriate water velocity and pressure head inside the penstock will determine the turbine speed and hence the generator speed but due to the fluctuation of water volume in the penstock pipe, the speed of the turbine/generator varies as well which affect generator rotation speed and frequency. In a normalapplication, the speed and frequency of the micro-hydro generator system must be maintained at a constant nominal value of 1500 RPM and frequency of 50 Hz but when connected to a local grid there is usually a tolerance of ±5%. So, in order to maintain this constant generator speed and frequency value, most micro-hydro turbine system regulates the rate of water discharge to the turbine which in turn change the turbine-generator speed by the use of speed governing system.

During the turbine operation when water discharge flow through a penstock pipe, the micro-hydro turbine system responses consists of two scenarios:

(a) The transient response: which is the initial state of the turbine-generator speed response caused by the initial pressure change due to the introduction of water flow to the turbine system? In this case, the system is unstable due to changes in turbine power and speed.
(b) The steady state response: when the turbine-generator system stabilizes and gives the steady-state values of speed and mechanical power

2.3.1 Penstock pipe modelling

A: Discharge *Q(s)* and Pressure head *H(s)*

Considering the water flow in the penstock pipe from the inlet as point 1 (water entrance) and the outlet as point 2 (water exit) in Figure 2.1 below, the volumetric flow rate in the penstock Q is given by;

$$Q = A_1 U_1 = A_2 U_2 \tag{1.1}$$

where; Q = water discharge (m^3/s), A = penstock cross-sectional area (m^2) and U = water flow velocity in the penstock (m/s), H or H_s = Water height (m) (In steady state operating condition, $H = H_s$ while during transient's condition $H \neq H_s$)

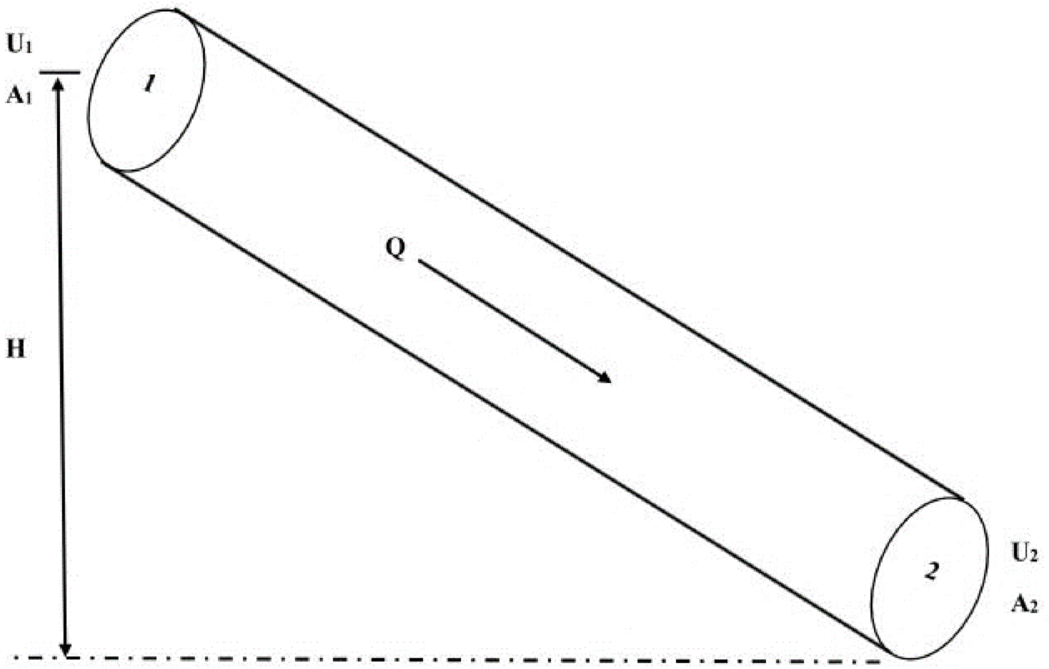

Figure 2.1: Water flow discharge through a penstock pipe

Water flowing in the penstock from the intake at point 1 and exit at point 2 and when considering the volume flow rate in the penstock, the amount of water flow at point 1 and point 2 will remains the same [4] and thus the water flow discharge Q can be written as:

$$Q = AU \qquad (1.2)$$

From the above equation, the value of Q can be interpreted as the rate of change volume discharge Δv (m^3) with a change in time Δt *(s)* and depends on the water flow velocity U and penstock cross section area A.

So applying Δv to the above equation, the change in volume flow of the water is given by:

$$\frac{\Delta v}{\Delta t} = AU \qquad (1.3)$$

Where: Δv= change in volume flow and Δt = change in time

Re-arranging the above equation gives, $\Delta v = AU\Delta t$and when differentiating with respect to time it gives, $dv = AU(t)dt$

Simplifying the above equation based on the water flow velocity in the penstock as a function of time, the change in volume flow Δv becomes:

$$\Delta v = AU(t)\Delta t \qquad (1.4)$$

On the other hand, the water flow in the penstock from the inlet and the exit can be related to the Bernoulli's theorem that:

$$\frac{U_1^2}{2g} + \frac{p_1}{\rho g} + H_1 = \frac{U_2^2}{2g} + \frac{p_2}{\rho g} + H_2 \qquad (1.5)$$

Since both the penstock inlet and exit are exposed to the atmospheric pressure, then $p_1 = p_2$ and also the velocity at the penstock inlet U_1 is usually very small compared to the velocity at the penstock exit U_2 due to large volume of water from the forebay to the penstock inlet, so in this case, $U_1$is set to be zero ($U_1 = 0$)

Thus, the above equation can be simplified and becomes;

$$\frac{-U_2^2}{2g} + 0 + (H_1 - H_2) = 0 \qquad (1.6)$$

Let H represent the difference between $H_1$and H_2 then the water flow exit velocity U_2 in the penstock can be expressed as:

$$U_2 = \sqrt{2gH} \qquad (1.7)$$

This is the exit velocity of the water in the penstock which is the function of pressure head H and gravity. This is valid for most micro-hydro turbine systems as the velocity of the water flow in the penstock depend on the pressure head H.

Consider the water volume flow between the inlet and exit of the penstock pipe, the total change in volume of water entering the penstock (Δv_{in}) during the change in time Δt should be equal to the change in volume of water exit from the penstock ($\Delta \upsilon_{exit}$). The above relation is based on the law of conservation of energy.

The equation $\Delta v = AU(t)\Delta t$ can be expanded to determine the volume of water entering and exit the penstock as follows:

$$\Delta \upsilon_{exit} = A_2 U_2(t)\Delta t \qquad (1.8)$$

But $A_2 = \frac{\pi {D_2}^2}{4}$ where D_2 = diameter of the penstock pipe at the exit point

Then; $$\Delta \upsilon_{exit} = \frac{\pi {D_2}^2}{4} \text{x} \sqrt{2gH} \text{ x } \Delta t \qquad (1.9)$$

When considering the $\Delta H(t)$ as the amount of water pressure head in the penstock at the incremental time Δt, then the rate of change water volume flow at the inlet $\Delta \upsilon_{inlet}$ is given by;

$$\Delta \upsilon_{inlet} = -A_1 \text{x} \Delta H(t) = -\left(\frac{\pi {D_1}^2}{4}\right) \text{x} \Delta H(t) \qquad (1.10)$$

The rate of volume at the penstock inlet should be equal to the rate of volume at the penstock exit, i.e. $\Delta\upsilon_{inlet} = \Delta\upsilon_{exit}$ which gives the following relation:

$$-\left(\frac{\pi D_1^2}{4}\right) x \Delta H(t) = \frac{\pi D_2^2}{4} x \sqrt{2gH} x \Delta t \quad (1.11)$$

This can be re-written as:

$$\frac{\Delta H(t)}{\Delta t} = -(H(t))^{1/2} x \left(\frac{D_2}{D_1}\right)^2 x \sqrt{2g} \quad (1.12)$$

Considering the continuous flow of water through the penstock which will give the rate of change in time, $\Delta t = 0$

So, the above equation can be written as;

$$\frac{dH(t)}{dt} = -\left(\frac{D_2}{D_1}\right)^2 x \sqrt{2gH(t)} \quad (1.13)$$

$$\frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 x \sqrt{2gH(t)} = 0 \quad (1.14)$$

The above equation can be considered for the ideal condition with no head loss in the penstock and it represents a first order differential equation of the rate of change of pressure head with time.

When considering the head losses on the system, the rate of water flow discharge per unit time on the penstock must be equal to the volumetric flow rate on the penstock, which gives:

$$\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 x \sqrt{2gH(t)} \quad (1.15)$$

But, $\sqrt{2gH(t)} = U(t)$

Which gives;

$$\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 x U(t) \quad (1.16)$$

The above water flow velocity with time $U(t)$ is the velocity at the inlet of the penstock;

$$U(t) = U_1(t)$$

In this case when considering the two water flow locations on the penstock inlet and exit and since water is continuously flowing on the system, so the inlet and exit water flow velocities may be represented as follows:

$A_1U_1(t) = A_2U_2(t)$ Substituting the value of U_1gives;

$$U_1(t) = \frac{A_2}{A_1}U_2(t) = \left(\frac{D_2}{D_1}\right)^2 x \ U_2(t) \quad (1.17)$$

Which can also be written as;

$$\frac{U_1(t)}{U_2(t)} = \left(\frac{D_2}{D_1}\right)^2 \quad (1.18)$$

Substituting the above $\frac{U_1(t)}{U_2(t)}$ values to the equation $\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 x \ U(t)$ will gives the following;

$$\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{U1^2(t)}{U_2(t)}\right) \quad (1.19)$$

The velocity $U_1(t)$represents the water flow velocity at the penstock inlet and the velocity $U_2(t)$ is the flow velocity at the penstock exit. Since the speed of the water flows in the penstock pipe will be determined by the approach velocity from the intake which is given by:

$$U_1(t) = U_1 = \sqrt{2gH(t)} \quad (1.20)$$

Substituting the $U_1 = \sqrt{2gH(t)}$value to the equation $\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{U1^2(t)}{U_2(t)}\right)$ gives the following;

$$\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \frac{1}{U_{2(t)}}2gH(t) \quad (1.21)$$

To maintain the constant speed of the turbine at a given time, the water flow velocity $U_2(t)$ at the exit of the penstock must be constant (U_2).

Thus, the above equation can be written as;

$$\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \frac{1}{U_2}2gH(t) \quad (1.22)$$

Taking Laplace transform of the above equation, assuming initial condition to be zero gives the transfer function as follows:

$$sQ(s) = sH(s) + \frac{1}{U_2}2gH(s) \quad (1.23)$$

Also, from the design formula, water flow discharge at the exit Q_2 is given by the following relation;

$$Q_2 = A_2U_2 = \frac{\pi D_2^2}{4}U_2 \quad (1.24)$$

In this case, considering the turbine design parameter values of $Q_2 = 0.45m^3/s$ and $D_2 = 460mm$

Putting these values to the above equation gives the following results for the water flow velocity in the penstock pipe:

$$U_2 = \frac{4Q}{\pi D_2^2} = 2.7 \text{ m/s}$$

Then, substituting the values of U_2 and using $g = 9.81 \text{ m/s}^2$ on the equation

$sQ(s) = sH(s) + \frac{1}{U_2}2gH(s)$ gives the following relation;

$$sQ(s) = sH(s) + \frac{2*9.81}{2.7}H(s) \quad (1.25)$$

$sQ(s) = sH(s) + 7.27H(s)$

$$sQ(s) = H(s)[s+7.27] \quad (1.26)$$

Re-arranging the above equation gives the following relation;

$$\frac{H(s)}{Q(S)} = \frac{s}{s+7.27} \quad (1.27)$$

This is the transfer function of the pressure head $H(s)$ to flow discharge $Q(s)$ in the penstock pipe which is represented in the following Figure 2.2 block diagram.

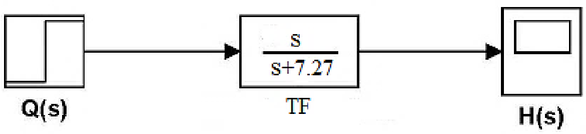

Figure 2.2: Step signal input Q(s) to the signal response H(s)

On the other hand, when considering the water flow velocity U inside the penstock pipe and in relation to gate opening G and pressure head H, the following equation applies with per unit values [5].

$$\bar{U} = \bar{G}\sqrt{\bar{H}} \quad (1.28)$$

For the non-linear model, the water flow velocity in the penstock can be very difficult to determine, but when substituting the value of velocity U with discharge Q value as $\bar{Q} = A\bar{U}$. In this case, the following condition applies for gate valve section: $0 \leq G \leq A$ and in per unit values this can be written as $0 \leq G \leq 1$ [6]

$$\bar{Q} = A\,\bar{G}\sqrt{\bar{H}} \quad (1.29)$$

$$\sqrt{\bar{H}} = \frac{\bar{Q}}{A\,\bar{G}}$$

where; $\bar{G} = A_t\bar{g}$ and A is the penstock area where water flows and given by $A = \frac{\pi D^2}{4} = 0.166$ m^2

D = diameter of the penstock = 460mm

Eliminating the normalized values gives;

$H = Q^2/(AG)^2 = \frac{Q^2}{G^2\,A^2} = (\frac{Q^2}{G^2})\frac{1}{A^2}$

Thus;

$$H = (\frac{Q^2}{G^2})\frac{1}{A^2} \quad (1.30)$$

The above equation can be represented in the following block diagram with the input gate valve position G and flow discharge Q and output pressure head H as shown in Figure 2.3 below.

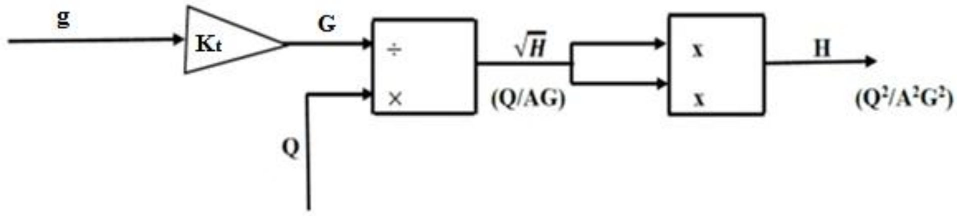

Figure 2.3: Formulation of hydraulic head H equation based on Q and G during modelling

B: Discharge *Q(s)* and Mechanical Power *P(s)*

The Power output per second (*P(t)*) from the hydropower plant depends on the water discharge, weight per second, pressure head and system efficiency. This relation is given by the following equation.

$$P(t) = mQ\eta H(t) \quad (1.31)$$

where; $P(t)$ = power per second, m = mass of water flow per second, Q = discharge and H = pressure head and η is the hydraulic efficient.

From the above formula the value of flow discharge Q and pressure head H are usually maintained at a constant value at the turbine entry so, in this case the weight of water flowing per second can be expressed in terms of water density ρ and water volume υ as follows:

$$P(t) = \rho vQ\eta H(t) \quad (1.32)$$

From the above equation the volume v per second is referred to as flow discharge, so the above equation can be re-written as:

$$P(t) = \rho Q^2\eta H(t)\ (W) \quad (1.32)$$

or in terms of kW capacity

$$P(t) = Q^2\eta H(t)\ (kW)$$

Re-arranging the above equation and taking the Laplace transform we get;

$$H(s) = \frac{P(s)}{\eta Q^2} \quad (1.34)$$

Substituting the value of $H(s)$ to the above equation from the transfer function of Equation 5.28 we get;

$$\frac{P(s)}{Q(s)} = \frac{(\eta Q^2)s}{s+7.27} \quad (1.35)$$

Applying the following system design parameters, flow discharge $Q = 0.45$ m^3/s and hydraulic turbine efficiency = 72% to the above equation gives the following Laplace equation;

$$\frac{P(s)}{Q(s)} = 0.146\frac{s}{s+7.27} \quad (1.36)$$

The following Figure 2.4 is the transfer function block diagram of turbine power *P(s)* to flow discharge *Q(s)* as shown in details below

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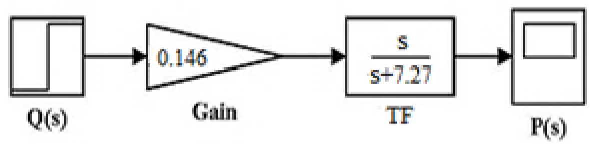

Figure 2.4: Block diagram of Q(s) signal input to P(s) signal output

It is noted that in this case there is a gain value of 0.146 on the transfer function for the flow discharge, *Q(s)* to the power output *P(s)* compared to a previous transfer function for *Q(s)* and *H(s)*.

When relating the input signals for pressure head *H(s)* and flow discharge *Q(s)*, the power output signal can be derived with the following relation based on mechanical power, head and flow in the turbine as follows:

For the transfer function that relates turbine velocity to the turbine head is given as [7];

$$TF(s) = \frac{\Delta\bar{U}}{\Delta\bar{H}} \quad (1.37)$$

But for a non-linear turbine model for this design where the pressure wave in the penstock and water compressibility is not considered, then the above equation becomes:

$$TF(s) = -\frac{1}{T_w s} \quad (1.38)$$

where; T_w= Water starting time (1.79 seconds)

From the equation 5.38, substituting the $\bar{U}$ with $\bar{Q}/A_p$ we get the following;

$$TF(s) = \frac{\Delta\bar{Q}/A_p}{\Delta\bar{H}} = -\frac{1}{T_w s} \quad (1.39)$$

where; A_p = Area of the penstock pipe

Simplifying the above equation, we get:

$$\Delta\bar{Q}\ \Delta\bar{H} = -\frac{A_p}{T_w s} \quad (1.40)$$

In relation to modelling, the value of A_P is taken as a gain (constant), so in this case, the mechanical power P_m developed in a micro-hydro turbine depends on the net pressure head ΔH and water flow discharge ΔQ.

Then the equation 1.41 below relates the turbine mechanical power P_m in with the net head ΔH and net flow discharge ΔQ as follows:

$$\bar{P}_m = \Delta\bar{Q}\ \Delta\bar{H} \quad (1.41)$$

But theoretically, power from the micro-hydro turbine due to water flow discharge is given by equation 1.12. So, in this case, the parameters ρg are the power gains for the system model and are represented as a constant value.

When considering per unit values of the mechanical power output, the nominal operating point of the hydro turbine need to be considered.

Thus, at the nominal operating point;

$$P_N = \rho g H(Q_N - Q_v) \text{ and also } Q_N = AG\sqrt{Hs} \quad (1.42)$$

where; P_N = Nominal power of turbine, H_s = Water height, Q_N= Nominal water flow,
Q_v= Volumetric flow losses.

When normalizing the above turbine power equation, we get:

$$P_m\,(p.u). = \frac{H}{Hs}\frac{Q-Q_v}{QN-Q_v} = \frac{H}{Hs}\frac{Q_N}{QN-Q_v}\frac{Q-Q_v}{N} \quad (1.43)$$

Simplifying the above equation, we get:

$$P_m\,(p.u) = K_P H_{pu}\,(Q_{pu} - Qv_{pu}) \quad (1.44)$$

In this case, the value K_p = power gain given by ρg in nominal values and $K_p = \frac{1}{1-Q_{v(p.u)}}$ in p.u. values.

The above Equation 1.44 with p.u. values can be simulated based on the set parameters as shown on the block diagram of Figure 2.5 below.

When turbine efficiency is considered during modelling then the output turbine power obtained is the mechanical power given by the following equation:

$$P_m = \rho g H \Delta Q \eta_t \quad (1.45)$$

The above equation 1.45 is the general equation used to calculate the potential power output from a hydro turbine system with known site parameters of head H and water flow discharge Q. The required amount of water flow is allowed to pass through the gate valve and discharged to the turbine and cause the turbine to rotate which produce hydraulic power. In the conversion process, the output hydraulic power value must be multiplied by a power gain which is ρg factor from the formula and also turbine efficiency η_t in order to obtain mechanical power as shown in Figure 2.5 below.

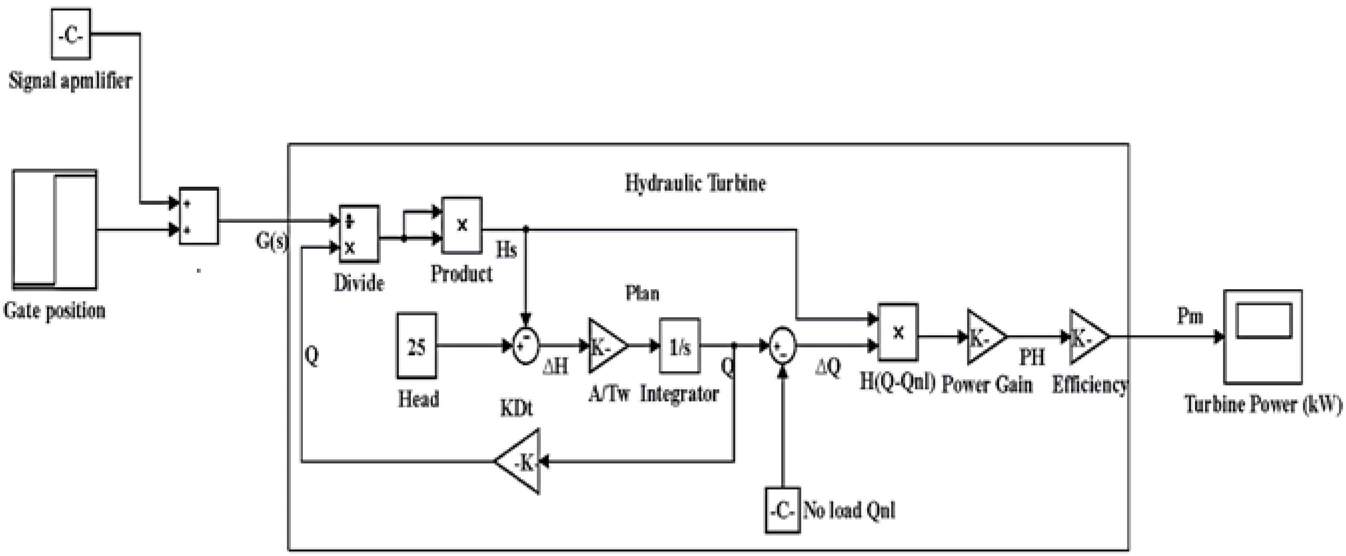

Figure 2.5: Penstock hydraulic model with power gain and efficient factor

2.3.2 Hydraulic Turbine modelling

In micro-hydro turbine modelling with a small variation around an operating point, consideration should be made on steady-state operating conditions and also on their linearized condition which can be related by the Taylor series linear equation as follows for the change in flow discharge and mechanical power respectively [8]:

$$\Delta Q = a_{11}\Delta H + a_{12}\Delta G + a_{13}\Delta\omega \quad (1.46)$$

$$\Delta P_m = a_{21}\Delta H + a_{22}\Delta G + a_{23}\Delta\omega \quad (1.47)$$

Where the constants:

a_{11} and a_{21} = partial derivative of discharge and mechanical power with per unit deviation in head
a_{12} and a_{22} = partial derivative of discharge and mechanical power with per unit deviation in gate position
a_{13} and a_{23} = partial derivative of discharge and mechanical power with per unit deviation turbine speed.

The above two equations are related to the small change of water flow discharge, ΔQ and change in mechanical power, ΔP_m with respect to change in water head ΔH, change in gate position ΔG and change in turbine speed $\Delta\omega$. Applying partial derivative to the above equations in relation to the penstock water flow dynamics, the transfer functions of the micro hydro turbine can be developed as follows:

Equation 1.46: $\Delta Q = a_{11}\Delta H + a_{12}\Delta G + a_{13}\Delta\omega$with partial derivative gives as follows:

$$a_{11} = \frac{\partial Q}{\partial H} and a_{12} = \frac{\partial Q}{\partial G} \quad and \ a_{13} = \frac{\partial Q}{\partial \omega}$$

Equation 7.47: $\Delta P_m = a_{21}\Delta H + a_{22}\Delta G + a_{23}\Delta\omega$

$$a_{21} = \frac{\partial P_m}{\partial H} and a_{22} = \frac{P_m}{\partial G} \quad and \ a_{23} = \frac{P_m}{\partial_\omega}$$

In this case, the penstock-turbine Laplace transform transfer function related to the change of mechanical power ΔP_m, to the change in gate position, ΔG is given by [9]:

$$\frac{\Delta P_m(s)}{\Delta G(s)} = \frac{a_{23} - (a_{13} \mathrm{x} a_{21} - a_{11} \mathrm{x} a_{23})sT_w}{1 + a_{11} sT_w} \tag{1.48}$$

When considering the hydraulic turbine change in mechanical power $\Delta P_m$to the change in gate valve position ΔG, the developed transfer function in the block diagram is given in Figure 2.6 below.

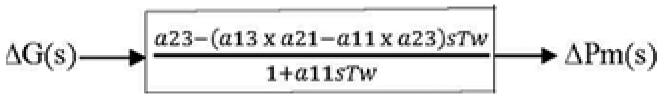

Figure 2.6: Block diagram transfer function of the linearized turbine model

To develop the system, transfer function we have to consider for an Ideal hydraulic turbine (lossless turbine) at rated speed and head with the initial values of partial derivatives as follows [6]:
$a_{11} = 0.5$, $a_{12} = 0$, $a_{13} = 1$

and

$a_{21} = 1.5$, $a_{22} = -1$, $a_{23} = 1$

Substituting the above partial derivative values for the ideal turbine we obtain the following short form linearized first-order transfer function:

$$\frac{\Delta P_m(s)}{\Delta G(s)} = \frac{1 - sT_w}{1 + 0.5sT_w} \tag{1.49}$$

The above Equation 1.49 has been represented in a block diagram as the deviation in gate position and mechanical power as shown in Figure 2.7 below.

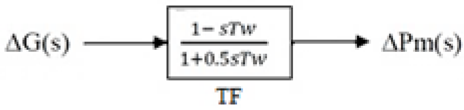

Figure 2.7: Stead state turbine-penstock transfer function block diagram

From the above Equation 1.49, it noted that the main characteristic of the transfer function is that behaves as a non-minimum phase system on the Laplace transform s-plane.

The term T_w is related to the micro-hydro turbine water starting time which depends on the speed of water flow in the penstock. It can be defined as the time required to accelerate water flow in the penstock pipe from the minimum speed at the water entering the penstock to a maximum speed of the water exit from the penstock.

Mathematically;

$$T_w = \frac{L \mathrm{\ x\ } Q}{H \mathrm{\ x} A_p \mathrm{\ x\ } g} = 1.79 \text{ seconds} \tag{1.50}$$

where; L = length of the penstock (162 m), Q = design discharge (0.45 m^3/s), H = design head (25 m), A_p = area of the penstock pipe (0.166 m^2) and g = acceleration due to gravity (9.81 m^2/s).

The value of $T_w$is an important parameter used in micro-hydropower modelling and this value usually varies because it depends on the speed (U) of the water flow in the penstock. Typical values of T_w for micro-hydropower systems range between 0.5 seconds to 4.0 seconds [6].

In this case, using the T_w value of 1.79 seconds for the turbine design and by referring to the equation 1.50 and Figure 2.8, the stead-state turbine-penstock transfer function for the change in gate valve position ΔG in relation to the change in mechanical power ΔP_m can be represented in the following block diagram. Also, in this case, an integrator and feedback control loop has been introduced on the model together with the signal filter as shown in Figure 2.8 below.

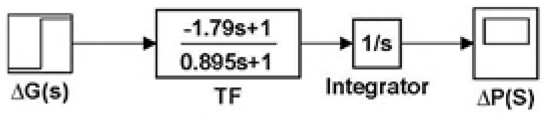

Figure 2.8: Block diagram for stead state turbine-penstock of a linearized micro hydro turbine model

On the other hand, the micro-hydropower systems with short and medium penstock length like that of Hhaynu river in Mbulu, the design consideration has been based on a non-linear system model, where the transfer function does not consider the travelling pressure wave and water compressibility, thus gives the simplified transfer function as follows [7]:

$$\frac{\Delta U(s)}{\Delta H(s)} = \frac{1}{sT_w} \tag{1.51}$$

Substituting the T_w value on the above equation gives:

$$\frac{\Delta U(s)}{\Delta H(s)} = \frac{1}{1.79s}$$

In hydraulic modelling, the relationship between power output signal *P(s)* to the water flow input signal *Q(s)* is obtained by substituting the values to the above equation 1.51 with *H(s)* values from equation 1.34 on which we obtain the following relation:

$$\frac{\Delta U(s)\eta Q^2}{P(s)} = \frac{1}{sTw} \qquad (1.52)$$

$$\frac{\Delta U(s)}{\Delta P(s)} = \frac{1}{\eta Q^2 \text{x } sTw} = \frac{1}{0.26s}$$

$$\frac{\Delta U(s)}{\Delta P(s)} = \frac{1}{0.26s} \qquad (1.53)$$

where; Q = 0.45 m^3/s, η = 0.72 and sT_w = 1.79 seconds.

This is the transfer function between the water flow velocity in the penstock to the power output from the turbine.

3. Conclusion

In the hydraulic turbine system model, the following parameters were used as modelling parameters to develop system transfer functions, water flow discharge, *Q* through the penstock pipe and pressure water head, *H* which have been obtained from the site measurements. The input parameter of hydraulic turbine system model is the water flow discharge/velocity through the penstock pipe with gating system to the turbine unit on which when modelled produce turbine power (Mechanical Power), P_m as output parameter. The modelling results shows that, the mechanical power produced is related to the amount of water flow discharge/velocity in the penstockpipe to the turbine system. Using the design parameters as modelling values for *Q* and *H*, the output turbine (mechanical) power, P_m obtained during modelling is 79.5 kW with the hydraulic turbine system efficiency, η_m of 72% with water starting time, T_W of 1.79 seconds.

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References

[1] NREL, *Small hydropower systems*. 2001, Department of Energy, USA: National Renewable Energy Laboratory.

[2] C, G., *Assessment of the Effect of Hydraulic Power Plants Governor Settings on Low-Frequency Inter-area Oscillations*. 2010, Middle East Technical University.

[3] Report, I.C., *Dynamic Models for Steam and Hydro Turbines in Power System Studies*, in *Power Apparatus and Systems*. Nov/Dec 1973. p. 1904 – 1915.

[4] Larock, B.E., Jeppson, R. W. and Watters, G. Z, *Hydraulics of Pipeline Systems*. 1999: CRC Press.

[5] Chaudhry, M.H., *Applied hydraulic transients*. 2nd ed. 1987, New York.

[6] Kundur, P., *Power System Stability and Control*. 1994, New York: McGraw-Hill.

[7] R. A. Naghizadeh, S.J.a.B.V., *Modelling Hydro Power Plants and Tuning Hydro Governors as an Educational Guideline.* International Review on Modelling and Simulations (I.RE.MO.S.), 2012. **5**(4).

[8] K., S.C., *Accurate low order model for hydraulic turbines-penstock.* IEEE Trans Energy Conversation, 1987(2): p. 196-200.

[9] IEEE, *Guide for the Application of Turbine Governing Systems for Hydroelectric Generating Units*, in *IEEE Standard* November 2004. p. 1207 - 2004.