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Ramanujan Sums of All Natural Numbers with Grandi Series

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Abstract: ***We are all know that the Ramanujan's theory of calculating the sum of all the natural number. When we came to known that sum of all-natural number is $\frac{-1}{12}$, that time everyone thinks about two things, one is how it possible that sum of positive number is negative & other is how it is very close to zero. Another research paper gives value of all natural number is $\frac{1}{12}$ then I started the study of this sum, then I find it is as zero. Quit interestingly but this value has more important than Ramanujan's sum because I did not consider the Grandi series for calculating this sum. And then it is very easy to calculate the sum of negative integer & sum of all odd natural number & sum of all even natural number.***

Keywords: Ramanujan, sum of natural numbers, negative value, Grandi series, research, odd natural number, even natural number

1. Introduction

We have to find the sum of all natural numbers 1+2+3+4+5+6+...........+∞ =?

Let s_1 = 1+2+3+4+5+6+……………+∞ (1)

s_2= 1-1+1-1+1-1+……………+∞ (2)

Equation (2) can be written as

s_2= 1- (1-1+1-1+1+……………+∞)

s_2 = 1- (s_2)

s_2+s_2 = 1

2s_2 = 1

s_2 = $^1/_2$ (3)

Let s_3 = 1-2+3-4+5-6+……………+∞ (4)

Subtracting equation (4) from (2) we have

($s_2 - s_3$) = 0+1-2+3-4+5……………..+∞

(s_2 –s_3) = S3

2s_3 = s_2

s_3 = $\frac{1}{2}s_2$

s_3= $\frac{1}{4}$ (5)

Now we take equation (1) & equation (3)

s_1 = 1+2+3+4+5+6+……………+∞

s_3 = 1-2+3-4+5-6+……………+∞

Subtract equation (3) from equation (1)

($s_1 - s_3$) = 4+8+12+16+20+24………

($s_1 - s_3$) = 4(1+2+3+4+5+6+……………+∞)

($s_1 - s_3$) = 4s_1

-s_3 =4$s_1 - s_1$ 3$s_1 = -s_3$

$s_1 = -\frac{1}{12}$

In this way ramanujan calculate the sum of infinite natural number, another way is a

s_1 = 1+2+3+4+5+6+……………+∞ (6)

On writing the terms of equation (6) leaving one space from the beginning

s_1 = +1+2+3+4+5+……………+∞ (7)

Add equation (6) and equation (7)

2s_1 = 1+3+5+7+9+11+…………..+∞ (8)

s_2 = 1-1+1-1+1-1+1-1+1-1+1……………+∞ (9)

(2s_1+s_2) = 2+2+6+6+10+10+14+14+ ⋯ ∞

(2s_1+s_2) = 4+12+20+28+…………..∞

(2s_1+s_2) = 4(1+3+5+7+…………..∞)

(2s_1+s_2) = 4(2s_1) (from equation 8)

(2s_1+s_2)) = 8s_1

6s_1 = s_2

s_2 = $^1/_2$

6s_1 = ½

S1 =1/12

In this is a way one researcher solve it both this two value close to what I obtained but different from both the values, this both sum discuss about the grandi series, this series never gives sum ½, it gives either 0 or 1. So I thought that lead the cause of error. I neglected this series & calculate the sum of natural numbers, as follow

Take

$s_1 = 1+2+3+4+5+6+7+8+9+\cdots$ (12)

$= (1+3+5+7+9+\cdots) + (2+4+6+8+10+\cdots)$

$= (1+3+5+7+9+\cdots)$ +2 $(1+2+3+4+5+6+7+8+\cdots)$

Second bracket of the above equation is the sum of natural number i.e. our equation (12)

$s_1 = (1+3+5+7+9+\cdots)$ +2(s_1)

Subtracting 2s_1 $from\ both\ sides$, we get

$s_1 - 2s_1 = 1+3++5+7+9+\cdots$

$-s_1 = 1+3++5+7+9+\cdots$ (13)

Now, equation (13) is defined the sum of negative integers is equal to the sum of all positive odd natural numbers another statement shall be stated as sum of all natural number is equal to the algebraic sum of the all negative odd integer. Now write the equation (13) as follow

$$-(s_1) = 1 + 3 + 5 + 7 + 9 + \cdots \quad (14)$$

Subtracting equation (12) from equation (14), we have

$$-(s_1) - s_1 = (1 + 3 + 5 + 7 + 9 + \cdots) - (1 + 2 + 3 + 4 + 5 + 6 + \cdots)$$

Subtracting values from second part term by, we get

$$-2s_1 = (0+1+2+3+4+5+6+7+8+9+10+\ldots)$$

$$-2s_1 = 0 + s_1$$

$$-3s_1 = 0 \quad (15)$$

Dividing equation (15) by -3 on both sides, we get

$s_1 = 0$

$$\text{i.e. } 1+2+3+4+5+6+7+8+\cdots = 0 \quad (16)$$

Now dividing equation (15) by 3, we get

$-s_1 = 0$

$$\text{i.e. } -1 - 2 - 3 - 4 - 5 - 6 - 7 - \cdots = 0 \quad (17)$$

This gives that algebraic sum of negative integer is zero.

Now consider the equation (13)

$$-s_1 = 1 + 3 + 5 + 7 + 9 + \cdots \quad (18)$$

By equation (17) it clears that

$$1+3+5+7+9+11 + \cdots = 0 \quad (19)$$

From equation (16), we have

$1+2+3+4+5+6+7+8+\cdots = 0$

This is also written as,

$$(1 + 3 + 5 + 7 + 9 + \cdots) + (2 + 4 + 6 + 8 + 10 + \cdots) = 0 \quad (20)$$

From equation (19), we can write equation (20) as,

$(0) + (2 + 4 + 6 + 8 + 10 + \cdots) = 0$

This become,

$$2 + 4 + 6 + 8 + 10 + \cdots = 0 \quad (21)$$

2. Result

1) $1+2+3+4+5+6+\cdots = 0$
2) $1+3+5+7+9+\cdots = 0$
3) $-1 - 2 - 3 - 4 - 5 - 6 - 7 - \cdots = 0$
4) $2 + 4 + 6 + 8 + 10 + \cdots = 0$

These are some sum of infinite numbers; interestingly all above sum gives answer '0'.

3. Conclusion

As we always say that sum of natural numbers is always greater than or equal to every number of that sum. But here this sum of infinite number contradicts to the stated statement, in this case sum of numbers in result (1) & (2) is less than every number of it, whereas in result (3) algebraic sum is greater than every number in that sum. Without using the grandi series such result we obtained so still one query arise why this happen?

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