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An Inventory Model with Partially Time-Varying Demand and Dynamic Cost Analysis

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Abstract: ***In this inventory model, I examine the inventory model with time-dependent demand that does not include item deterioration. Only time and demand have an impact on inventory level fluctuations. This system does not take shortages into account either. This model analyzes the effects of various parameters on ordering and holding costs. Evaluating the overall cost and examining the substantial impact on profit is the primary goal of this model's research. Numerical considerations that illustrate the nature of the cost in relation to varying levels of parameter variation are used to validate the model.***

Keywords: Inventory, demand, holding cost, ordering cost

1. Introduction

To expedite the creation of specific inventory, inventory management is one of the most important aspects of business. Good management may help the sector create a clear strategy, track leadership, and communicate freely, which will help businesses advance in the current neck-cutting climate. By controlling the distribution of resources, customer happiness, and personal growth, management may increase productivity and creativity. Sound financial management techniques guarantee robustness, and ongoing development leads to long-term success. Businesses can ignore risks, take advantage of opportunities, and achieve a standing development in any dynamic and competitive environment by using optimization concepts. The economy of businesses and the industry's level of market capture are greatly influenced by the volume of inventories. Keeping up with significant demand and increasing inventory sales to maximize profit in a difficult environment are challenges associated with storage. The primary determinants of market demand are product utility and visibility. In business, showcasing products can be advantageous. Product availability and display are key factors in raising public interest. Everybody is aware that industrial products always deteriorate and have a limited lifespan. Fruits and vegetables have fairly short lifespans, yet items like medications and technology devices have extensive lifespans. To ensure optimal usage and prevent perishable inventory, managers must pay close attention. A supply and demand chain system, dynamic environments, predicting market trends, and cutting down on waste may all be balanced with the help of effective inventory management. To improve industry policies, numerous researchers have created various inventory models. An inventory model for an exponentially decaying inventory system was created by researchers Ghare and Schrader [9]. An EOQ model with Weibull distribution deterioration has been provided by Covert and Philip [7]. An inventory policy that takes into account the trade credit financing environment system was published by Haley and Higgin [12]. A generalized EOQ inventory model for objects with deterioration as a weibull pattern has been provided by researcher Philip [19] in the early phases of inventory development. Additionally, given a linear demand pattern, Donaldson [8] provides inventory replenishment policy. He has also used numerical analysis to validate the model. A comment on an order level inventory model for a system with a constant rate of deterioration was published by Aggarwal [1]. Numerous deterministic inventory models have been examined by Bhunia and Maiti [2] while taking changing production rates into account. An EOQ model for degrading commodities with a linear demand trend and inventory shortages for all cycles has been examined by Chakrabarti and Chaudhuri [3]. An order level inventory model for a system with a constant rate of deterioration has been devised by Shah and Jaiswal [20]. An ordering policy for degrading commodities with acceptable shortages and payment delays has been provided by Jamal, Sarkar, and Wang [15]. An EOQ model for degrading items with time-varying demand and partial backlog has been established by Chang and Dye [4]. The resilient simulation-optimization approach for pre-disaster multi-period location-allocation-inventory planning has been the subject of an article by Ghasemi and Damghani [10]. A study on current developments in degradation inventory modeling was published by Goyal and Giri [11]. An optimal ordering interval strategy with known demand for commodities with varying rates of deterioration has been provided by Sharma and Kumar [21]. Shorages are also permitted in the model. An ideal ordering interval strategy with known demand for commodities with varying rates of deterioration and shortages has also been provided by Sharma and Kumar [22]. An article by Ouyang, Wu, and Cheng [18] describes an inventory model that includes decaying items with exponentially decreasing demand and partial backlogs. Inventory models for degrading items with power from stock-dependent demand have been provided by Teng, Cheng, and Ouyang [23]. An inventory model for economic order quantity policy for perishable goods under stock-dependent selling rate and time-dependent partial backlog has been developed by Chun-Tao Chang, Goyal, and Jinn-Tsair Teng [5]. An order level lot size inventory model for degrading products with exponentially declining demand has been reported by Mehta, Niketa, and Shah [17]. An essay on figuring out the best selling price and lot size with a variable rate of degradation and exponential partial backlog was written by Chung-Yuan Dye, Tsu-Pang Hsieh, and Liang-Yuh Ouyng [6]. An EOQ model for deteriorating items with time-dependent demand under inflationary conditions was presented by Jaggi and Mittal [13]. When end demand is both price and credit period sensitive, Jaggi, Kausar, and Khanna [14] have proposed a two-stage credit policy that optimizes retailers' unit selling price and fixed cycle duration. An application of the EOQ model with nonlinear holding cost to inventory management of perishables items has been noted by Mark, Vaidy, and Gilvan [16].

Here, we created an EOQ model with time-varying demand and no inventory degradation. We have computed the total expenses on various fields within a predetermined cycle time in this inventory model and examined the differences caused by all parameters on the overall cost. This model has been verified through numerical analysis.

2. Assumption and Notation

1) C_1 = Setup cost per unit item.
2) C_2 = Holding cost per unit item per unit time.
3) D(t) = demand = $\begin{cases} t^2 d, & 0 < t < t_1 \\ d, & t_1 \le t \le T \end{cases}$.
4) d = demand parameter for items.
5) T = Fixed cycle time.
6) Q = Initially inventory Level.
7) I(t) = Inventory level at time t.
8) $C(t_1, T)$ = Total cost per cycle time.
9) TD = Total demand in cycle time.
10) O = Ordering cost per order per unit.
11) H = Holding cost in cycle time.
12) TC = Total cost in cycle time.

3. Mathematical Model

We are looking at a perpetual inventory system that runs continuously and has variable demand up until a certain point in time, after which it remains constant until the end of the cycle. Deterioration of the inventory is not permitted by our policy. Let I(t) stand for the inventory level at any given time t. At time t = 0, inventory production began. The start of production marks the start of the procurement, administration, and sale processes, among other operations. If manufacturing stops during a cycle time, the inventory level quickly drops to zero at time T. The mathematical policy for a system is as follows:

$$\frac{d}{dt} I(t) = t^2 d, \qquad 0 \le t \le t_1 \qquad ...(1)$$

$$\frac{d}{dt} I(t) = d + I(t), t_1 \le t \le T \qquad ...(2)$$

The boundary conditions are $I(0) = 0,\ I(T) = Q$.

Solution of differential equation (1) is $I(t) = \frac{t^3}{3} d$

Solution of differential equation (2) is

$$I(t) = (dt + c_2)(1 + t + t^2)$$

After using the boundary condition

$$I(t) = -dt^2 + Q\left(1 - T + \frac{T^2}{2}\right)$$

From the above two solutions, we find

$$Q = \frac{d}{2} t_1^2 (3 + 2t_1^2)(2 + 2T + T^2)$$

The total demand in cycle time [0, T] is given as

$$D = \int_0^T D(t)dt \qquad = \frac{d}{6}(3T^2 - t_1^2)$$

Number of items in duration [0, T]

$$= \int_0^T I(t)dt = \frac{d}{6}\left\{(t_1^3 - T^3)(t_1^2 - 2t_1^2 + T + t_1^2 T) + Tt_1(4 - 5T + t_1)\right\}$$

Now we calculate the different cost related to this inventory model

Ordering cost =

$$QC = C_1 \frac{d}{2} t_1^2 (3 + 2t_1^2)(2 + 2T + T^2)$$

Holding cost =HC = $C_2 \int_0^T I(t)dt$

$$= \frac{C_2 d}{6}\left\{(t_1^3 - T^3)(t_1^2 - 2t_1 + T + t_1^2 T) + Tt_1(4 - 5T + t_1)\right\}$$

The total cost per unit time per unit item is given as

$$C(t_1, T) = \frac{1}{T}(\text{Ordering cost + Holding cost})$$

$$= C_1 \frac{d}{2T} t_1^2 (3 + 2t_1^2)(2 + 2T + T^2) + \frac{C_2 d}{6T}\left\{(t_1^3 - T^3)(t_1^2 - 2t_1 + T + t_1^2 T) + Tt_1(4 - 5T + t_1)\right\}$$

We now use the optimality criterion on cost to determine the best solution $\frac{\partial C(t_1, T)}{\partial t_1} = 0 = \frac{\partial C(t_1, T)}{\partial T}$. And determine stationary values of t_1 and T as t_1^*, T^*. At these stationary points, we find $\frac{\partial^2 C(t_1, T)}{\partial t_1^2}, \frac{\partial^2 C(t_1, T)}{\partial T^2}, \frac{\partial^2 C(t_1, T)}{\partial t_1 \partial T}$.

If $\frac{\partial^2 C(t_1, T)}{\partial t_1^2} \times \frac{\partial^2 C(t_1, T)}{\partial T^2} > \left(\frac{\partial^2 C(t_1, T)}{\partial t_1 \partial T}\right)^2$ then the cost will be minimum.

4. Numerical Example with Graph

We study the effect of different parameters on total cost,

Table 1

d	t_1	T	Cost
37	1.75	4.2	96.33
37	1.75	4.25	105.39
37	1.75	4.3	118.31
37	1.75	4.35	135.36
37	1.75	4.4	157.68
37	1.75	4.45	181.97
37	1.75	4.5	211.29
37	1.75	4.55	258.78
37	1.75	4.6	302.24
37	1.75	4.65	361.38

Table 2

d	t_1	T	cost
37	1.75	4.2	96.33
39	1.75	4.2	99.36
41	1.75	4.2	103.98
43	1.75	4.2	109.11
45	1.75	4.2	117.32
47	1.75	4.2	124.32
49	1.75	4.2	131.56
51	1.75	4.2	138.61
53	1.75	4.2	147.39
55	1.75	4.2	155.28

Table 3

d	t_1	T	cost
37	1.75	4.2	96.33
50	1.78	5	106.84
50	1.81	5	111.54
50	1.84	5	114.25
50	1.87	5	115.52
50	1.9	5	115.98
50	1.93	5	116.21
50	1.96	5	116.84
50	1.99	5	117.68
50	2.02	5	118.36

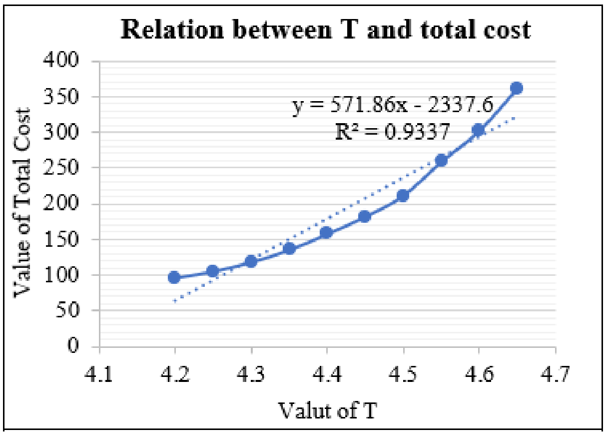

Figure 1

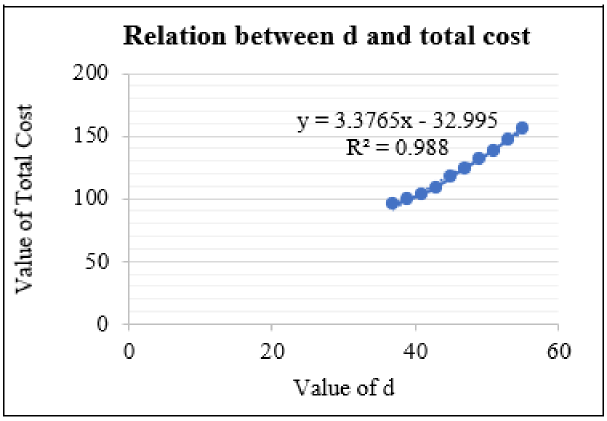

Figure 2

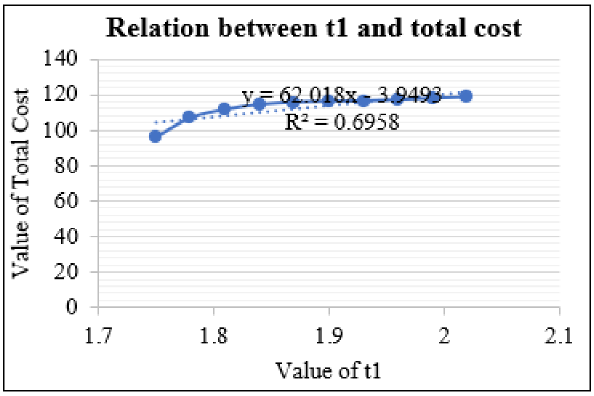

Figure 3

5. Conclusions and Remarks

The rate at which things are manufactured to restock the study's inventory is proportionate to the demand's fluctuations during a brief period of time after it stabilizes. The direct relationship between production rate and market demand satisfaction is evident. Demand suggests the importance of customer satisfaction. In the numerical example, these values are at the lowest level. The previous numerical calculation and graph analysis clearly show that the overall cost increases as d and T increase but lowers as t1 grows. This lowers the total cost. Given the facts, we could enhance our inventory system and generate substantial revenues. These factors have a major influence on production, cost, sales, and profitability in real-world scenarios. A number of factors, such as government taxes, labor expenses, workforce size, and equipment infrastructure, affect how productive our operations are. There are other things that could affect our inventory system. These may be incorporated into further study.

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