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Research on Solving Geometric Problems in College Entrance Examination Mathematics—Take the College Entrance Examination Questions from 2018 to 2022 as an Example

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Abstract: ***Analytic geometry is one of the most important and difficult points in the college entrance examination. This paper probes into the analytic geometry in the national paper of mathematics for college entrance examination from 2018 to 2022, and mainly analyzes the conic curve in the analytic geometry in the national paper of mathematics for college entrance examination from horizontal and vertical aspects. In this paper, the real problems in the past 5 years are evaluated from five aspects: content level, background level and reasoning level. Based on Polliats problem-solving theory, this paper studies the problems of trajectory and its equations, fixed-value and fixed-point problems, maximum-value and range problems, and existence problems, provide a reference for teachers teaching.***

Keywords: College entrance examination, Analytic geometry, Conic section**.**

1. Introduction of the Problem

This paper aims to study the analysis of conic sections in the college entrance examination mathematics, with the purpose of summarizing and exploring knowledge in this field, enhancing problem-solving skills, and laying a foundation for future research. Conic sections are an inevitable part of college entrance examination mathematics, characterized by their high flexibility. If students can overcome this challenge, they will achieve better results in the exam. Most articles on conic sections in college entrance examination mathematics have not delved deeply into these topics, lacking specialized research on such questions. Moreover, conic sections typically involve a large amount of knowledge and a broad scope, often including trigonometric functions, inequalities, and plane vector concepts. They place high demands on high school students thinking abilities, creativity, and computational skills. Therefore, I hope that the discussion in this paper will provide some insights into solving this issue.

2. Research Design

2.1 Research Methods

Research method is a method to express the internal laws of things, discover new things or have new insights. This paper mainly adopts the literature research method, statistical analysis method and Polya problem solving theory.

2.2 Subjects of Study

Based on a classification and in-depth study of relevant literature on conic sections in the analysis of geometry in the college entrance examination mathematics over the past five years, it has been found that there are many papers studying conic sections in the college entrance examination, but relatively few targeted studies have been conducted on recent years exam questions, and these studies are not comprehensive enough. Moreover, conic sections hold a significant position in college entrance examination mathematics. Therefore, this paper takes the national college entrance examination mathematics from 2018 to 2022 as its research object, combining the “Senior High School Mathematics Curriculum Standards” and the “Sichuan College Entrance Examination Syllabus,” to conduct a categorized study on conic section-related questions in the national college entrance examination mathematics. The research content mainly includes the following three aspects.

1) In the analysis of geometric problems in the national college entrance examination mathematics test from 2018 to 2022, data analysis was carried out in the form of statistical data.

2) In-depth and detailed comments were made on the proposition form, knowledge points, ideas and methods, and difficulty level of the analytic geometry questions in the national college entrance examination from 2018 to 2022. Bao Jiansheng was used [32] The professors difficulty coefficient model analyzes the difficulty of conic section examination in the national college entrance examination mathematics paper in the past five years;

3) Analyzed the relevant questions on conic sections in the national college entrance examination mathematics from 2018 to 2022, summarizing and categorizing them into trajectory problems and trajectory equation problems, fixed points and values problems, maximum and minimum value and range problems, as well as existence problems. For these four major test points, typical questions were selected to analyze problem-solving strategies and summarize solution methods.

3. Research Results

3.1 Frequency Analysis of Each Test Point of Conic

Sections

The so-called horizontal comparison refers to comparing data using years as the thread and specific years as units. In recent years, research and design on problem-solving questions in college entrance examination papers have significantly increased. The author has statistically analyzed conic section problems from the national college entrance examination mathematics papers for the years 2018-2022 and found that these problems can be categorized into several typical types, including solution problems, which include angle problems, coordinate problems, parameter problems, and curve equation problems. Calculation problems involve length, area, maximum and minimum values, and eccentricity. In proof problems, the main issues include fixed values, fixed points, and the validity of equations. These problem patterns are related to circles, hence they are referred to as "conic section" problems. Specific data is provided below Table 1:

Table 1

		18 years	19 years	20 years	21 years	22 years	amount to
solve	The Angle Problem	2					2
	Coordinate problem	1	1	2			4
	Parameter problem	6	6	6	4	4	26
	The curve problem	5	4	6	3	5	23
count	Length issues	3	2		3	2	10
	The area issue		3	3	2		8
	The problem of maximum value	1	2	2	3	2	10
	eccentricity	4	5	4	2	2	17
prove	Targeted problems		3	1		2	6
	The problem of valuation	2	3	2	1		8
	The equation holds	2					2
amount to		26	29	26	18	17	116

The analysis of the above table can get the following information:

From the number of conic section questions examined each year, there were 26 in 2018,29 in 2019,26 in 2020,18 in 2021, and 17 in 2022. Since the first three years had three sets of papers, and the last two years had two volumes, the number of questions examined in 2019 was the highest at 29. The other years had an average number of questions. For more details, see Figure 1.

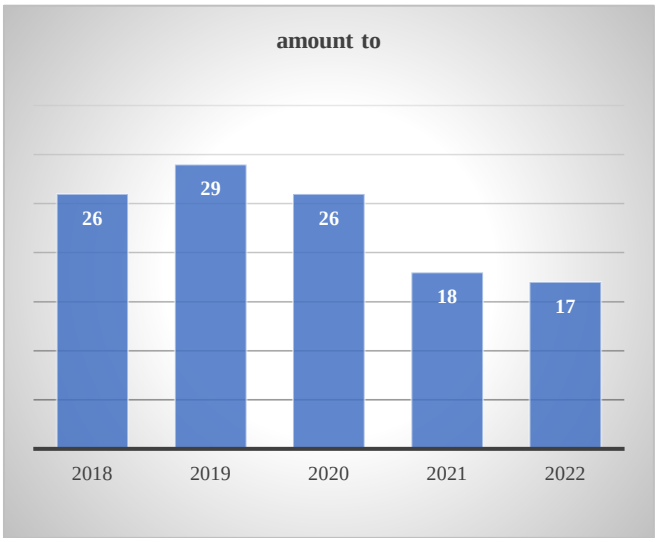

Figure 1

In terms of the concentration and dispersion of question types, 2019 showed the most diverse questioning methods, followed by 2018, with 2020 being the least. Parameter problems are a must-exam question type every year, with 26 instances over the past five years. Curve problems and eccentricity problems rank second and third, with 23 and 17 instances respectively. For specific distribution, see Figure 2.

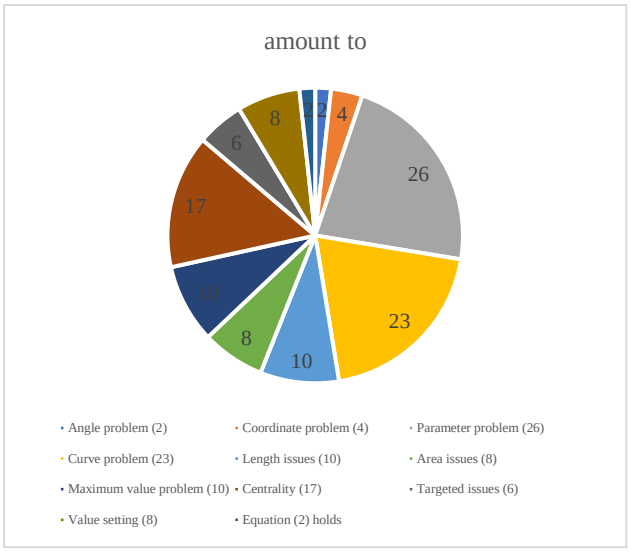

Figure 2

3.2 Comprehensive Difficulty Analysis of Conic Curve Questions

What are the differences in the difficulty of questions on different test papers? In what aspects does the difficulty reflect? What are the characteristics of the difficulty? "There needs to be a quantitative standard," said Bao jiansheng [32] Professor established a comprehensive mathematical test difficulty analysis model in "A Comparative Study on the Comprehensive Difficulty of Junior High School Mathematics Courses in China and the UK," dividing it into content level, background level, reasoning level, computational level, and knowledge integration level. This model has the ability to reflect the relationship between test difficulty and students acceptance capabilities. Professor Bao

Jianshengs theory of comprehensive difficulty analysis for mathematical tests simplifies test difficulty by breaking it down into five elements, then defining and assigning values to each element to meet different levels of difficulty requirements. This analytical method has significant advantages over traditional examination methods. It involves organizing the explanation process of test questions, calculating the difficulty value through computation, thus ensuring a certain degree of stability.

3.2.1 Comprehensive difficulty model

Through extensive reading of various literature and summarization, we combined Bao Jiansheng and Qiu Yating [9]. The difficulty model was derived and the comprehensive level model was obtained. See for details Table 2 Comprehensive difficulty model-It is divided into five horizontal factors: background level, content level, reasoning level, operation level and knowledge synthesis level.

Table 2: Comprehensive difficulty model

Difficulty indicators	Difficulty sub-indicators	explain	assignment
Background level	Background free		1
	Background of life and production	Based on life and production	2
	Scientific background	In a scientific context	3
Content level	memorize	The memory of concepts, formulas, rules and properties has the characteristics of mechanical and less connection	1
	understand	The understanding and application of learned theories, methods and processes, including the selection of knowledge and methods, the flexible application of procedural knowledge, and the establishment of links between different knowledge points	2
The level of reasoning	No reasoning	No need for reasoning	1
	Simple reasoning	It takes only one or two steps of reasoning	2
	Complex reasoning	It requires a multi-step reasoning process	3
Computational level	No operation	No calculations are required	1
	Simple calculations	Only simple symbolic numerical operations are required	2
	Complex calculations	Complex symbol operations and complex numerical operations are required	3
Comprehensive level of knowledge	One knowledge point	There is a point of knowledge involved in solving the problem	1
	Two points of knowledge	There are two points of knowledge involved in the problem solving process	2
	Multiple knowledge points	There are many knowledge points involved in solving problems	3

According to the five difficulty factors of the comprehensive difficulty coefficient model, certain weight coefficients are assigned at different levels. Generally, natural values are assigned to each level, that is, natural numbers are used for assignment. The difficulty coefficients of each difficulty factor in the test paper are calculated using the formula, where represents the 5 different dimensions, and represents the weight of the level at the dimension. $1,2,3\cdots\ d_i = \frac{\sum_{j=1}^{n} n_{ij}d_{ij}}{n}\left(\sum n_{ij} = n, i = 1,2,3\cdots\right)\ i = 1,2,3,4,5\ d_{ij}ij$

3.2.2 Comprehensive analysis of question level

Based on the difficulty model, I conducted a difficulty analysis of five levels of factors: “background,” “content,” “inference,” “calculation,” and “knowledge integration.” By analyzing the number of questions in the national college entrance examination mathematics papers from 2018 to 2022, I identified and summarized patterns in the proposition characteristics of conic section problems. The number of conic section questions tested each year is limited, so it is possible to analyze these patterns over the past five years.

1) Background level

To Table 3, the following information can be obtained from the analysis:

In terms of the number of exam questions, National Paper I, II, and III all included 37 background-free questions and 1 question related to life or production backgrounds, with no questions involving scientific backgrounds. Both National Paper II and National Paper III had a total of 31 questions, none of which involved life, production, or science. From the distribution across National Papers I, II, and III, it is evident that in the examination of conic sections, most questions directly presented knowledge without much connection to life or science, as shown below Figure 3.

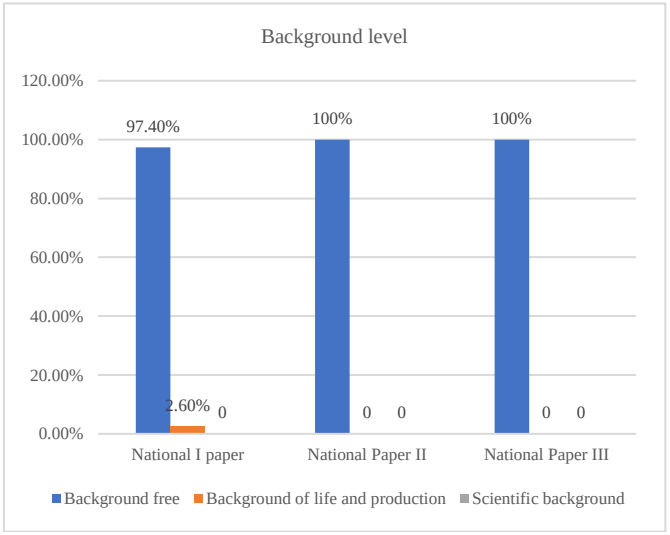

Figure 3

Table 3

Difficulty factors	Background free		Background of life and production		Scientific background		degree of difficulty
Level of grade	Number of questions	percentage	Number of questions	percentage	Number of questions	percentage	
National I paper	37	97.4%	1	2.6%	0	0%	1.03
National Paper II	31	100%	0	0%	0	0%	1
National Paper III	31	100%	0	0%	0	0%	1

In terms of difficulty, the difficulty coefficient for National Paper I is 1.03; the difficulty coefficients for National Paper II and National Paper III are both 1. Although the difficulty coefficients differ, all three papers are part of the national unified examination. In terms of background level assessment, compared to the three sets of papers, National Paper I is relatively richer. For more details, please refer to Figure 4.

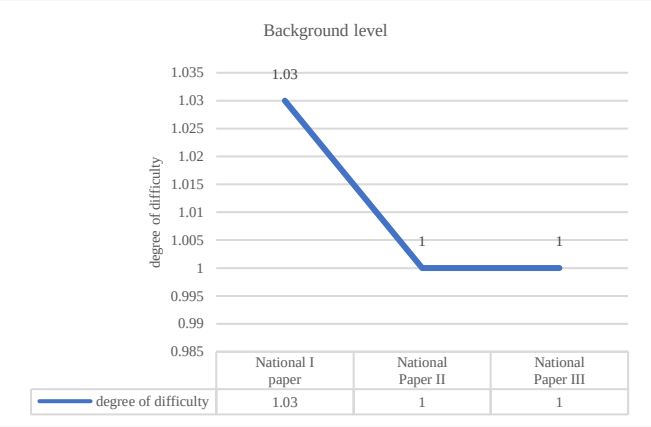

Figure 4

2) Content level

Table 4

Difficulty factors / Level of grade	memorize: Number of questions	memorize: percentage	understand: Number of questions	understand: percentage	degree of difficulty
National I paper	18	40.9%	26	59.1%	1.59
National Paper II	18	40.9%	26	59.1%	1.59
National Paper III	14	34.2%	27	65.8%	1.66

The following information can be obtained from Table 4.

From the perspective of the number of content levels examined in National Papers I, II, and III, National Paper I and II focus on memorization with 18 questions and comprehension with 26 questions; National Paper III focuses on memorization with 14 questions and comprehension with 27 questions. Overall, when examining conic sections, more emphasis is placed on students ability to apply knowledge and adaptability. In terms of content level, National Paper III has fewer questions on memorization compared to National Papers I and II, but it has more questions on comprehension. See the table below:

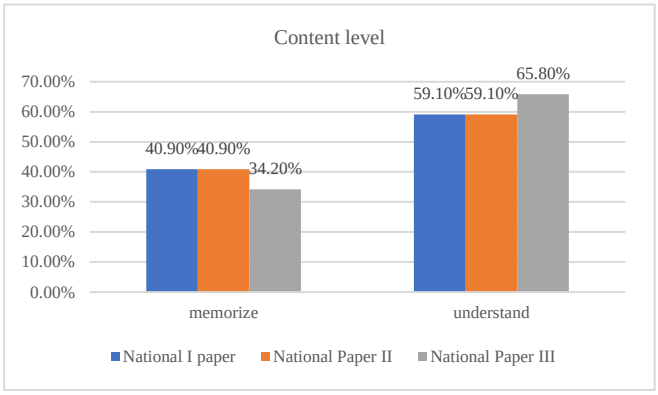

Figure 5

From the perspective of difficulty coefficient, the difficulty coefficient of national I paper and national II paper is 1.59, and the difficulty coefficient of national III paper is 1.66. The examination difficulty of national III paper is higher than that of national I paper and national II paper, and shows an increasing trend year by year, indicating that the requirements for students knowledge points are getting higher and higher. For specific data, please refer to the following Figure 6:

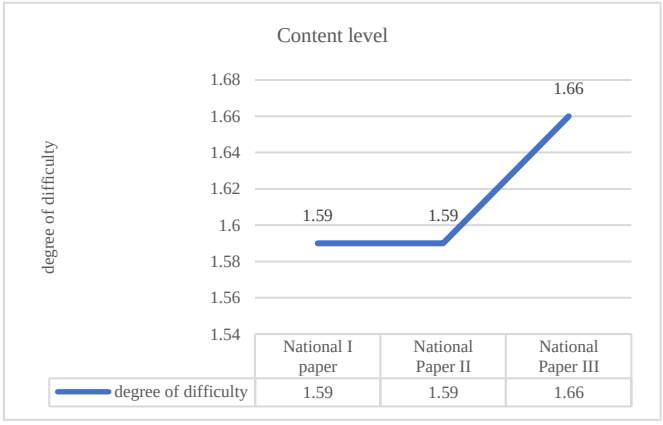

Figure 6

3) The level of reasoning

Table 5

Difficulty factors / Level of grade	No reasoning: Number of questions	No reasoning: percentage	Simple reasoning: Number of questions	Simple reasoning: percentage	Complex reasoning: Number of questions	Complex reasoning: percentage	degree of difficulty
National I paper	0	0%	14	40%	21	60%	2.60
National Paper II	0	0%	18	52.9%	16	47.1%	2.47
National Paper III	0	0%	20	58.8%	14	41.2%	2.41

Analyze Table 5 The following information can be obtained:

From the perspective of the number of reasoning questions in National Papers I, II, and III, Paper I has 14 simple reasoning questions and 21 complex reasoning questions; Paper II has 18 simple reasoning questions and 16 complex reasoning questions; Paper III has 20 simple reasoning questions and 14 complex reasoning questions. The number of questions in Papers I, II, and III that do not involve reasoning is zero. This indicates that in the examination of conic sections, greater emphasis is placed on students reasoning skills. The number of simple reasoning questions shows an increasing trend in Papers I, II, and III, while the number of complex reasoning questions shows a decreasing trend. See the table below:

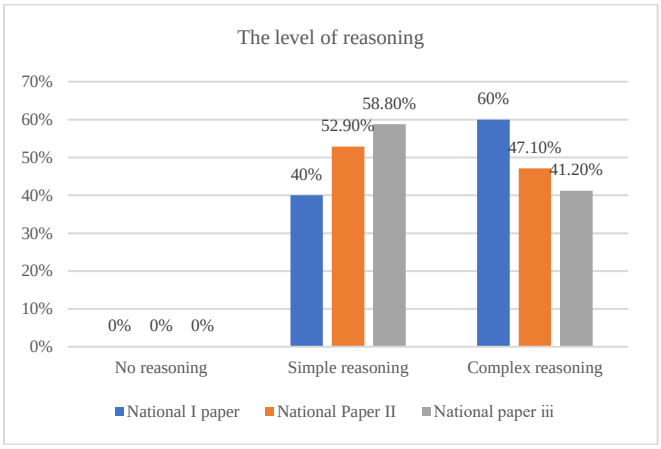

Figure 7

From the perspective of difficulty levels, the difficulty coefficient of National Paper I is 2.6, National Paper II is 2.47, and National Paper III is 2.41. The difficulty coefficient shows a downward trend. This indicates that in the examination of conic sections, the ranking of difficulty is: National Paper I> National Paper II> National Paper III. Compared to National Paper II and National Paper III, National Paper I places higher demands on students reasoning abilities. For more details, see the table below:

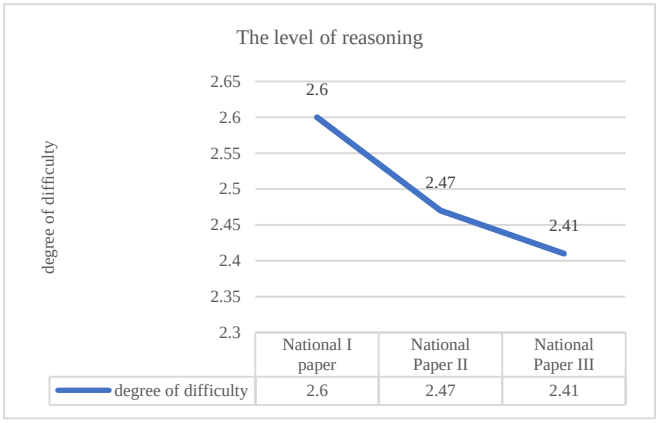

Figure 8

4) Computational level

Table 6

Difficulty factors / Level of grade	No operation Number of questions	No operation percentage	Simple calculations Number of questions	Simple calculations percentage	Complex calculations Number of questions	Complex calculations percentage	degree of difficulty
National I paper	0	0%	12	26.7%	33	73.3%	2.73
National Paper II	0	0%	15	34.9%	28	65.1%	2.65
National Paper III	0	0%	16	35.6%	29	64.4%	2.64

Table 6 provides the following information:

From the perspective of the number of computational problems in the National Paper I, II, and III exams, the National Paper I, II, and III all have zero computational questions in conic section examinations. In the National Paper I exam, there are 12 simple computational problems and 33 complex computational problems. The National Paper II has 15 simple computational problems and 28 complex computational problems; the National Paper III has 16 simple computational problems and 29 complex computational problems. Compared to the three papers, the number of similar computational problems is not significantly different. The National Paper I places higher demands on computational skills, being stronger than the National Paper II and III; while the National Paper II and III show very little difference in the percentage of simple and complex computational problems among the questions. Specific data are shown in the table below:

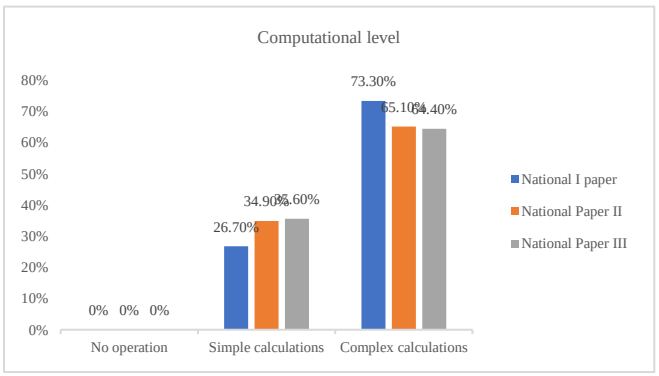

Figure 9

Observing the difficulty coefficients of conic section problems in the national college entrance examination, it can be found that the difficulty coefficient for National Paper I is the highest at 2.73, followed by National Paper II at 2.65, and National Paper III at 2.64, showing a downward trend. Therefore, it can be concluded that from the perspective of conic section examination, National Paper I places more emphasis on students computational skills and has higher requirements compared to National Paper II and National Paper III. The specific data is as follows Figure 10 As shown:

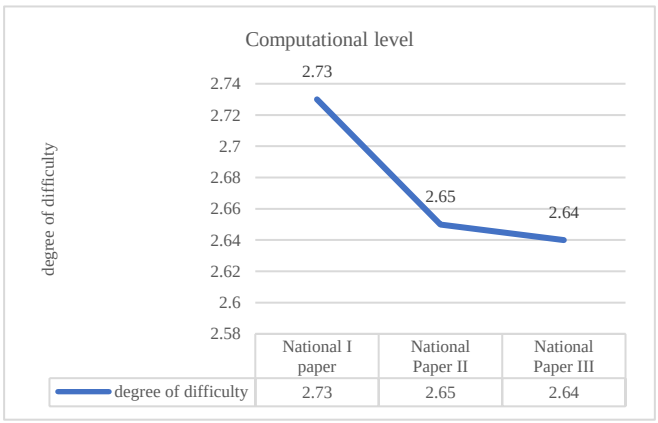

Figure 10

5) Comprehensive level of knowledge

Table 7

Difficulty factors / Level of grade	One knowledge point Number of questions	One knowledge point percentage	Two points of knowledge Number of questions	Two points of knowledge percentage	Multiple knowledge points Number of questions	Multiple knowledge points percentage	degree of difficulty
National I paper	1	2.3%	8	18.2%	35	79.5%	2.77
National Paper II	2	4.5%	13	29.5%	29	66.0%	2.61
National Paper III	1	2.4%	12	28.6%	29	69.0%	2.67

To the above Table 7 The analysis can yield the following information:

Regarding the number of comprehensive knowledge level questions in National Paper I, II, and III, one knowledge point has 1 question, two knowledge points have 8 questions, and multiple knowledge points have 35 questions. In National Paper II, one knowledge point has 2 questions, two knowledge points have 13 questions, and multiple knowledge points have 29 questions; in National Paper III, only one knowledge point has 1 question, two knowledge points have 12 questions, and multiple knowledge points have 29 questions. Although all three papers cover multiple knowledge points, the number of questions for each knowledge point is relatively low. Additionally, the exams emphasize the assessment of comprehensive application skills. Among them, National Paper I places more emphasis on the comprehensiveness of questions. Compared to National Paper II and III, National Paper I poses more challenges for students to integrate and convert knowledge points. The percentage difference in the number of questions assessing two and multiple knowledge points between National Paper II and III is minimal, as shown below Figure 11:

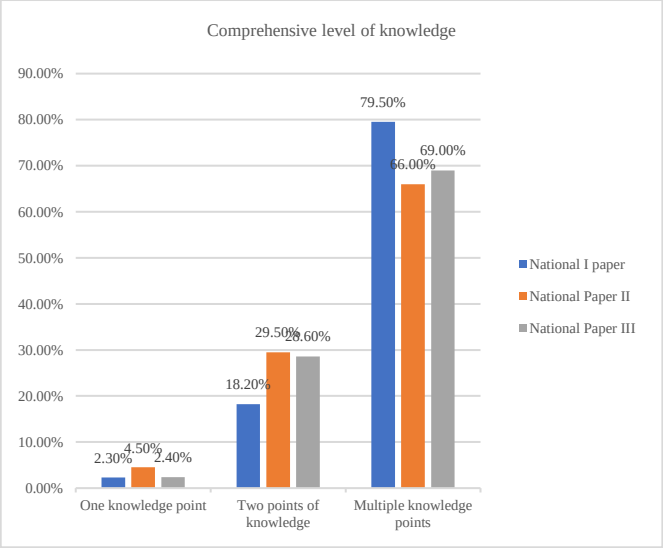

Figure 11

The difficulty coefficient of the national college entrance examination shows a downward trend. Specifically, the difficulty coefficient of the National I Paper is the highest at 2.77, followed by the National III Paper with a difficulty coefficient of 2.67, and lastly, the National II Paper with a difficulty coefficient of 2.61. In terms of conic section examinations, the ranking is National I Paper> National III Paper> National II Paper. For more details, see below Figure 12:

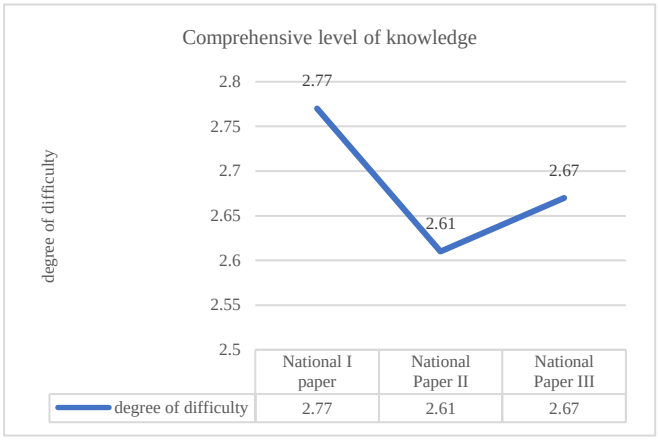

Figure 12

Project Fund

1. Key Laboratory of Computational Physics, Sichuan Provincial Universities and Colleges, Open Fund Project in 2024 “Practical Research on Optimizing the Design of High school Mathematics Homework and Activities under the Background of New Textbooks”, No. YBUJSWL-YB-2024-06.

2.Yibin University School of Mathematics and Physics Teaching Reform Project, Project Number: 156-22030001.

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