DOI: 10.53469/jrve.2025.07(02).01

Exploring Rogers-Ramanujan-Type Identities Modulo 9, 11, 18, and 22

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Abstract: ***In this paper, some identities of Rogers_Ramanujan Type related to modulo 9, 11, 18 and 22 is derived with the incorporation of generalized Bailey pairs and some standard results established by Andrew V. Sills [1] using some q − difference relations.***

Keywords: Rogers-Ramanujan Type Identities, Jacobi's Triple Product Identity, Bailey Pairs

Mathematics Subject Classification: 11P84, 11P81, 33D15, 05A17

1. Introduction

We begin by recalling the famous Rogers-Ramanujan Identities:

The Rogers-Ramanujan Identities:

For |q|<1,
$\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q;q)_n} = \prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0,2,3 \pmod 5$

and,
$\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q;q)_n} = \prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0,1,4 \pmod 5$
where $(q;q)_n = (1-q)$, for n≥1

which are known as the celebrated original Rogers-Ramanujan Identity. These two identities have motivated extensive research over the past hundred years. These identities are due to L.J. Rogers [5] and were rediscovered independently by S. Ramanujan [7] and I. Schur [4]. In 1940's W.N. Bailey undertook a careful study of Rogers work and greatly simplified into two papers [8] and [9]. In these papers, Bailey proved some more generalized formula that helps to find more identities of Rogers-Ramanujan Type.

Definitions 1.1:
For |q|<1, the q-shifted factorial is defined by

$$(a;q)_0 = 1$$

$(a;q)_n = \prod_{k=0}^{n-1}(1-aq^k)$, for n≥1
and $(a;q)_\infty = \prod_{k=1}^{\infty}(1-aq^k)$.

It follows that $(a;q)_n = \frac{(a;q)_\infty}{(aq^n;q)_\infty}$

The multiple q-shifted factorial is defined by

$$(a_1,a_2,\ldots,a_m;q)_n = (a_1;q)_n(a_2;q)_n\ldots(a_m;q)_n$$
$$(a_1,a_2,\ldots,a_m;q)_\infty = (a_1;q)_\infty(a_2;q)_\infty\ldots(a_m;q)_\infty.$$

<u>Jacobi's Triple Product Identity:(see [3] 2.2.10 and 2.2.11)</u>

$(zq^{\frac{1}{2}}, z^{-1}q^{\frac{1}{2}}, q:q)_\infty = \sum_{n=-\infty}^{\infty}(-1)^n z^n q^{n^2/2}$ (1.1)

And its corollary

$$\sum_{n=-\infty}^{\infty}(-1)^n q^{(2k+1)\frac{n(n+1)}{2}-in} = \sum_{n=-\infty}^{\infty}(-1)^n q^{(2k+1)\frac{n(n+1)}{2}-in}(1-q^{(2n+1)i})$$
$=\prod_{n=0}^{\infty}(1-q^{(2k+1)(n+1)})(1-q^{(2k+1)n+i})(1-q^{(2k+1)(n+1)-i})$ (1.2)

Definition 1.2: A pair of sequences $(\alpha_n(a,q), \beta_n(a,q))$ is called a Bailey pair if for $n \geq 0$,
$\beta_n(a,q) = \sum_{r=0}^{n}\frac{\alpha_r(a,q)}{(q,q)_{n-r}(aq,q)_{n+r}}$ (1.3)
In [8] and [9], Bailey proved the following result known as "Bailey Lemma".

Bailey's Lemma: If $(\alpha_r(a,q), \beta_j(a,q))$ form a Bailey pair, then

$$\frac{1}{(\frac{aq}{\rho_1};q)_n(\frac{aq}{\rho_2};q)_n}\sum_{j\geq 0}\frac{(\rho_1;q)_j(\rho_2;q)_j(\frac{aq}{\rho_1\rho_2};q)_{n-j}}{(q;q)_{n-j}}\left(\frac{aq}{\rho_1\rho_2}\right)^j\beta_j(a;q) = \sum_{r=0}^{n}\frac{(\rho_1;q)_r(\rho_2;q)_r}{(\frac{aq}{\rho_1};q)_r(\frac{aq}{\rho_2};q)_r(q;q)_{n-r}(aq;q)_{n+r}}\left(\frac{aq}{\rho_1\rho_2}\right)^r\alpha_r(a;q)$$ (1.4)

Some important consequences of this lemma are the following corollary: ([see [1], Eqn. (9), (10))

Corollary 1.1: If $(\alpha_m(a,q), \beta_j(a,q))$ form a Bailey pair, then
$\sum_{j\geq 0}a^j q^{j^2}\beta_j(a,q)=\frac{1}{(aq;q)_\infty}\sum_{m=0}^{\infty}a^m q^{m^2}\alpha_m(a,q)$ (1.5)

$\sum_{j\geq 0}a^j q^{j^2}(-q;q^2)_j\beta_j(a,q^2)=$
$\frac{(-aq;q^2)_\infty}{(aq^2;q^2)_\infty}\sum_{m=0}^{\infty}\frac{a^m q^{m^2}(-q;q^2)_m}{(-aq;q^2)_m}\alpha_m(a,q^2)$ (1.6)

In [8] and [9], Bailey considered several Bailey pairs which are special cases of a more general Bailey pair involving additional parameters d and k.

Parameterized Bailey pair:

Let $\lambda = -\frac{3}{2}d^2 + dk + \frac{1}{2}d$, $h = |\frac{2\lambda}{d}|$, and $t = d + h + 2$.

Let $\alpha_{d,k,m}(a,q) = \begin{cases} \frac{(-1)^r a^{(k-d)r} q^{\left(dk-d^2+\frac{d}{2}\right)r^2-\frac{d}{2}r}(aq^{2d};q^{2d})_r(a;q^d)_r}{(a;q^{2d})_r(q^d;q^d)_r} \\ 0 \quad if\ m = dr, and\ otherwise \end{cases}$

and

$$\beta_{d,k,m}(a,q) = \begin{cases} \lim_{r\to 0} \frac{{}_{t+1}W_t\,(a;\gamma_1,\gamma_2,\dots,\gamma_h,\mu_1,\mu_2,\dots,\mu_d;q^d;\tau^h a^{k-d}q^{nd})}{(a,aq;q)_n} & if\ \lambda \ge 0 \\ \lim_{r\to 0} \frac{{}_{t+1}W_t\,(a;\delta_1,\delta_2,\dots,\delta_h,\mu_1,\mu_2,\dots,\mu_d;q^d;\frac{a^{k-d}q^{nd}}{\tau^h})}{(a,aq;q)_n} & if\ \lambda < 0 \end{cases}$$

where $\gamma_j = \frac{q^{\lambda/h}}{\tau}$, $\mu_j = q^{d-j-n}$, $\delta_j = \tau a q^{d-\lambda/h}$.

$${}_{s+1}W_s\,(a_1;a_4,a_5,\dots,a_{s+1};q;z) = {}_{s+1}\phi_s \begin{bmatrix} a_1, qa_1^{1/2}, -qa_1^{1/2}, a_4, \dots, a_{s+1}; q, z \\ a_1^{1/2}, -a_1^{1/2}, \frac{qa_1}{a_4}, \dots, \frac{qa_1}{a_{s+1}} \end{bmatrix},$$

and,

$${}_{s+1}\phi_s \begin{bmatrix} a_1, a_2, \dots, a_{s+1}; q, z \\ b_1, b_2, \dots, b_s \end{bmatrix} = \sum_{r=0}^{\infty} \frac{(a_1,a_2,\dots,a_{s+1};q)_r}{(q,b_1,b_2,\dots,b_s;q)_r}.\ z^r.$$

Then $\alpha_{d,k,m}(a,q)$ and $\beta_{d,k,m}(a,q)$ form a Bailey pair. Bailey considered the special cases $\alpha_{d,k,m}(a,q)$ for $(d,k) = (1,2),(2,2),(2,3)\,and\,(3,4)$ in [8]. Each of these four (d,k) sets is particularly nice, as the resulting expression for $\alpha_{d,k,m}(a,q)$ is summable by Jackson's theorem ([2],238, $eqn(II-20)$). Thus, $\beta_{d,k,m}(a,q)$reduces to a finite product, and upon substituting it in (1.5) the left hand side of the resulting $a - RRT$identity will be a single-fold sum.

Definition 1.3: For $k \ge 1$, and $1 \le i \le k$,

$$Q_{d,k,i}(a) = Q_{d,k,i}(a,q) = \frac{1}{(aq;q)_\infty}\sum_{n\ge 0} \frac{(-1)^n a^{kn} q^{\left(dk+\frac{d}{2}\right)n^2+\left(k-i+\frac{1}{2}\right)dn}(1-a^i q^{(2n+1)di})(aq^d;q^d)_n}{(q^d;q^d)_n}$$
(1.7)

In [1], Andrew V. Sills has derived the following results with incorporation of the parameterized Bailey pairs and some q-difference equations as noted in [1].

Theorem 1.1: The following q −difference equation is valid: (See [1], eqn.(19) and (20))

$Q_{d,k,1}(a,q) = \frac{1}{(aq;q)_{d-1}}\ Q_{d,k,k}(aq^d,q)$ (1.8) *and for* $2 \le i \le k$,

$Q_{d,k,i}(a,q) = Q_{d,k,i-1}(a,q) + \frac{a^{i-1}q^{(i-1)d}}{(aq;q)_{d-1}}\ Q_{d,k,k-1}(aq^d,q)$
(1.9)

Theorem 1.2: For $i = 1,2,3,4$ (*see [1], Theorem 3.16, p. 19*])

$F_{3,4,i}(a,q) = Q_{3,4,i}(a,q)$ (1.10)

where,

$$F_{3,4,1}(a,q) = \sum_{n=0}^{\infty} \frac{a^n q^{n^2+3n}\,(aq^3;q^3)_n}{(aq;q)_{2n+2}(q;q)_n}$$

$$F_{3,4,2}(a,q) = \sum_{n=0}^{\infty} \frac{a^n q^{n^2+2n}\,(aq^3;q^3)_n}{(aq;q)_{2n+2}(q;q)_n}$$

$$F_{3,4,3}(a,q) = \sum_{n=0}^{\infty} \frac{a^n q^{n^2+n}\,(aq^3;q^3)_n}{(aq;q)_{2n+1}(q;q)_n}$$

$$F_{3,4,4}(a,q) = \sum_{n=0}^{\infty} \frac{a^n q^{n^2}\,(a;q^3)_n}{(a;q)_{2n}(q;q)_n}$$

Theorem 1.3: For $i = 1,2,3,4,5$ (*see [1], Theorem 3.19, p. 20*])

$F_{3,5,i}(a,q) = Q_{3,5,i}(a,q)$ (1.11)

where $F_{3,5,1}(a,q) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} \frac{a^{n+r}q^{n^2+3r^2+3n+3r}(aq^3;q^3)_{n-r}}{(aq;q)_{2n+2}(q;q)_{n-3r}(q^3;q^3)_r}$

$F_{3,5,2}(a,q) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} \frac{a^{n+r}q^{n^2+3r^2+3n+3r}(aq^3;q^3)_{n-r}\,(1+aq^{3r+3})}{(aq;q)_{2n+2}(q;q)_{n-3r}(q^3;q^3)_r}$

$F_{3,5,3}(a,q) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} \frac{a^{n+r-1}q^{n^2+3r^2-3}(a;q^3)_{n-r}\,(q^{3r}+aq^{6r+3}-1)}{(a;q)_{2n}(q;q)_{n-3r}(q^3;q^3)_r}$

$F_{3,5,4}(a,q) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} \frac{a^{n+r-1}q^{n^2+3r^2+3r}(a;q^3)_{n-r}}{(a;q)_{2n}(q;q)_{n-3r}(q^3;q^3)_r}$

$F_{3,5,5}(a,q) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} \frac{a^{n+r-1}q^{n^2+3r^2}(a;q^3)_{n-r}}{(a;q)_{2n}(q;q)_{n-3r}(q^3;q^3)_r}$

2. We derive some transformations from (1.10) and (1.11) which will be used in obtaining Identities related to Rogers-Ramanujan Type:

Setting $i = 1,2,3,4$ successively in (1.10) and then replacing $q\ by\ q^{1/3}\ and\ q^{2/3}$ respectively for each this particular value of$i$, we obtain the following eight transformations:

$$\sum_{n=0}^{\infty} \frac{a^n q^{(n^2+3n)/3}\,(aq;q)_n}{(aq^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_n} = \frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge 0} \frac{(-1)^n a^{4n} q^{(9n^2+7n)/2}(1-aq^{2n+1})(aq;q)_n}{(q;q)_n} \quad (2.1)$$

$$\sum_{n=0}^{\infty} \frac{a^n q^{(2n^2+6n)/3}\,(aq^2;q^2)_n}{(aq^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_n} = \frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge 0} \frac{(-1)^n a^{4n} q^{(9n^2+7n)}(1-aq^{4n+2})(aq^2;q^2)_n}{(q^2;q^2)_n} \quad (2.2)$$

$$\sum_{n=0}^{\infty} \frac{a^n q^{(n^2+2n)/3}\,(aq;q)_n}{(aq^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_n} = \frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge 0} \frac{(-1)^n a^{4n} q^{(9n^2+5n)/2}(1-a^2q^{4n+2})(aq;q)_n}{(q;q)_n} \quad (2.3)$$

$$\sum_{n=0}^{\infty} \frac{a^n q^{(2n^2+4n)/3}\,(aq;q)_n}{(aq^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_n} = \frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge 0} \frac{(-1)^n a^{4n} q^{(9n^2+5n)}(1-a^2q^{8n+4})(aq^2;q^2)_n}{(q^2;q^2)_n} \quad (2.4)$$

$$\sum_{n=0}^{\infty} \frac{a^n q^{(n^2+n)/3}\,(aq;q)_n}{(aq^{1/3};q^{1/3})_{2n+1}(q^{1/3};q^{1/3})_n} = \frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge 0} \frac{(-1)^n a^{4n} q^{(9n^2+3n)/2}(1-a^3q^{(6n+3)})(aq;q)_n}{(q;q)_n} \quad (2.5)$$

$$\sum_{n=0}^{\infty}\frac{a^n q^{(2n^2+2n)/3}\,(aq^2;q^2)_n}{(aq^{2/3};q^{2/3})_{2n+1}(q^{2/3};q^{2/3})_n}=\frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{4n}q^{(9n^2+3n)}(1-a^3q^{(12n+6)})(aq^2;q^2)_n}{(q^2;q^2)_n}\quad(2.6)$$

$$\sum_{n=0}^{\infty}\frac{a^n q^{n^2/3}\,(a;q)_n}{(aq^{1/3};q^{1/3})_{2n}(q^{1/3};q^{1/3})_n}=\frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{4n}q^{(9n^2+n)/2}(1-a^4q^{(8n+4)})(aq;q)_n}{(q;q)_n}\quad(2.7)$$

$$\sum_{n=0}^{\infty}\frac{a^n q^{2n^2/3}\,(a;q^2)_n}{(a;q^{2/3})_{2n}(q^{2/3};q^{2/3})_n}=\frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{4n}q^{(9n^2+n)}(1-a^4q^{(16n+8)})(aq^2;q^2)_n}{(q^2;q^2)_n}\quad(2.8)$$

In similar way, setting $i=1,2,3,4,5$ successively in (1.11) and then replacing q *by* $q^{1/3}$ *and* $q^{2/3}$ respectively for each particular value of i, we obtain the following ten transformations:

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r}q^{(n^2+3r^2+3n+3r)/3}(aq;q)_{n-r}}{(aq^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+9n)/2}(1-aq^{2n+1})(aq;q)_n}{(q;q)_n}\quad(2.9)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r}q^{(2n^2+6r^2+6n+6r)/3}(aq^2;q^2)_{n-r}}{(aq^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+9n)}(1-aq^{4n+2})(aq^2;q^2)_n}{(q^2;q^2)_n}\quad(2.10)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r}q^{(n^2+3r^2+3n+3r)/3}(aq;q)_{n-r}\,(1+aq^{r+1})}{(aq^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}$$
$$=\frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+7n)/2}(1-a^2q^{(4n+2)})(aq;q)_n}{(q;q)_n}\quad(2.11)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r}q^{(2n^2+6r^2+6n+6r)/3}(aq^2;q^2)_{n-r}\,(1+aq^{2r+2})}{(aq^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}$$
$$=\frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+7n)}(1-a^2q^{(8n+4)})(aq^2;q^2)_n}{(q^2;q^2)_n}\quad(2.12)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(n^2+3r^2-3)/3}(a;q)_{n-r}\,(q^r+aq^{2r+1}-1)}{(a;q^{1/3})_{2n}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}$$
$$=\frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+5n)/2}(1-a^3q^{(6n+3)})(aq;q)_n}{(q;q)_n}\quad(2.13)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(2n^2+6r^2-6)/3}(a;q^2)_{n-r}\,(q^{2r}+aq^{4r+2}-1)}{(a;q^{2/3})_{2n}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}$$
$$=\frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+5n)}(1-a^3q^{(12n+6)})(aq^2;q^2)_n}{(q^2;q^2)_n}\quad(2.14)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(n^2+3r^2+3r)/3}(a;q^{1/3})_{n-r}}{(a;q^{1/3})_{2n}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+3n)/2}(1-a^4q^{(8n+4)})(aq;q)_n}{(q;q)_n}\quad(2.15)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(2n^2+6r^2+6r)/3}(a;q^2)_{n-r}}{(a;q^{2/3})_{2n}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+3n)}(1-a^4q^{(16n+8)})(aq^2;q^2)_n}{(q^2;q^2)_n}\quad(2.16)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(n^2+3r^2)/3}(a;q)_{n-r}}{(a;q^{1/3})_{2n}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(aq^{1/3};q^{1/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+n)/2}(1-a^5q^{(10n+5)})(aq;q)_n}{(q;q)_n}\quad(2.17)$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(2n^2+6r^2)/3}(a;q^2)_{n-r}}{(a;q^{2/3})_{2n}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\frac{1}{(aq^{2/3};q^{2/3})_\infty}\sum_{n\ge0}\frac{(-1)^n a^{5n}q^{(11n^2+n)}(1-a^5q^{(20n+10)})(aq^2;q^2)_n}{(q^2;q^2)_n}\quad(2.18)$$

3. Main Results

3.1 Rogers-Ramanujan Type Identities Modulo 9:

Setting $a=1,q$ successively in the transformations (2.1), (2.3) and (2.5) respectively and then using (1.1), the following identities of Rogers-Ramanujan Type is found:

$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(n^2+3n)/3}\,(q;q)_n}{(q^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_n}=\frac{1}{(q;q)_\infty}\sum_{n=0}^{\infty}(-1)^n q^{\frac{9n^2+7n}{2}}(1-q^{2n+1})$$
$$=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{9n^2+7n}{2}}$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n},$$

where $n\not\equiv 0,1,8 \pmod 9$ (3.1.1)

$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(n^2+6n+9)/3}\,(q;q)_{n+1}}{(q;q^{1/3})_{2n+3}(q^{1/3};q^{1/3})_n}=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{9n^2+n}{2}}+q^3\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{9n^2+15n}{2}}\right)$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^3\prod_{n=0}^{\infty}\frac{1}{1-q^n}\quad(3.1.2)$$

where $n\not\equiv 0,4,5 \pmod 9$ and $n\not\equiv 0,3,6 \pmod 9$

$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(n^2+2n)/3}\,(q;q)_n}{(q^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_n}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{9n^2+5n}{2}}$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n},$$

where $n \not\equiv 0,2,7 \pmod 9$ (3.1.3)

$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(n^2+5n)/3}\,(q;q)_{n+1}}{(q;q^{1/3})_{2n+3}(q^{1/3};q^{1/3})_n}=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+3n)/2}+q^2\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+13n)/2}\right)$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^2\prod_{n=0}^{\infty}\frac{1}{1-q^n}\ (3.1.4)$$
where $n \not\equiv 0,3,6 \pmod 9$ and $n \not\equiv 0,2,7 \pmod 9$

$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(n^2+n)/3}\,(q;q)_n}{(q^{1/3};q^{1/3})_{2n+1}(q^{1/3};q^{1/3})_n}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+3n)/2}$$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0,3,6 \pmod 9$ (3.1.5)

$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(n^2+4n+3)/3}\,(q^2;q)_n}{(q;q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_n}=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+5n)/2}+q\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+11n)/2}\right)$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q\prod_{n=0}^{\infty}\frac{1}{1-q^n}\ (3.1.6)$$
where $n \not\equiv 0,2,7 \pmod 9$ and $n \not\equiv 0,1,8 \pmod 9$

Also, setting $a=q$ in the transformation (2.7), we find
$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(n^2+3n)/2}\,(q;q)_n}{(q^{4/3};q^{1/3})_{2n}(q^{1/3};q^{1/3})_n}=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+\prod_{n=0}^{\infty}\frac{1}{1-q^n}\ (3.1.7)$$
where $n \not\equiv 0 \pmod 9$ and $n \not\equiv 0,1,8 \pmod 9$

3.2 Rogers-Ramanujan Type Identities Modulo 11:

Setting $a=1,q$ successively in the transformations (2.9), (2.11), (2.13), (2.15) and (2.17) respectively, we find the following identities of Rogers-Ramanujan Type:
$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2+3n+3r)/3}(q;q)_{n-r}}{(q^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\prod_{n=0}^{\infty}\frac{1}{1-q^n},$$
where $n \not\equiv 0,1,10 \pmod{11}$ (3.2.1)
$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2+6n+6r+12)/3}(q;q)_{n-r+1}}{(q;q^{\frac{1}{3}})_{2n+3}(q^{\frac{1}{3}};q^{\frac{1}{3}})_{n-3r}(q;q)_r}$$
$$=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{11n^2+n}{2}}+q^4\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+19n)/2}\right)$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^4\prod_{n=0}^{\infty}\frac{1}{1-q^n}\ (3.2.2)$$
where $n \not\equiv 0,5,6 \pmod{11}$ and $n \not\equiv 0,4,7 \pmod{11}$
$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2+3n+3r)/3}(q;q)_{n-r}\,(1+q^{r+1})}{(q^{1/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+7n)/2}$$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0,2,9 \pmod{11}$ (3.2.3)
$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2+6n+6r+9)/3}(q^2;q)_{n-r}\,(1+q^{r+2})}{(q^{4/3};q^{1/3})_{2n+2}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}$$
$$=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{11n^2+3n}{2}}+q^3\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{11n^2+17n}{2}}\right)$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^3\prod_{n=0}^{\infty}\frac{1}{1-q^n}\ (3.2.4)$$
where $n \not\equiv 0,4,7 \pmod{11}$ and $n \not\equiv 0,3,8 \pmod{11}$
$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2-3)/3}(q;q)_{n-r-1}\,(q^r+aq^{2r+1}-1)}{(q^{1/3};q^{1/3})_{2n-1}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+5n)/2}$$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0,3,8 \pmod{11}$ (3.2.5)
$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2+3n+3r)/3}(q;q)_{n-r}\,(q^r+q^{2r+2}-1)}{(q;q^{1/3})_{2n}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}$$
$$=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{11n^2+5n}{2}}+q^2\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{11n^2+15n}{2}}\right)$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^2\prod_{n=0}^{\infty}\frac{1}{1-q^n}\ (3.2.6)$$
where $n \not\equiv 0,3,8 \pmod{11}$ and $n \not\equiv 0,2,9 \pmod{11}$
$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2+3r)/3}(q^{1/3};q^{1/3})_{n-r-1}}{(q^{1/3};q^{1/3})_{2n-1}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+3n)/2}$$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0,4,7 \pmod{11}$ (3.2.7)
$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2+3n+6r)/3}(q;q^{1/3})_{n-r}}{(q;q^{1/3})_{2n}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{11n^2+7n}{2}}+q\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{11n^2+13n}{2}}\right)$$
$$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q\prod_{n=0}^{\infty}\frac{1}{1-q^n}\ (3.2.8)$$

where $n \not\equiv 0, 2, 9 (\text{mod } 11)$ and $n \not\equiv 0, 1, 10$ (mod 11)

$$\frac{(q^{1/3};q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(n^2+3r^2)/3}(q;q)_{n-r-1}}{(q^{1/3};q^{1/3})_{2n-1}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+n)/2}$$

$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0, 5, 6$ (mod 11) (3.2.9)

$$\frac{(q;q^{1/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(n^2+3r^2+3n+3r-3)/3}(q;q)_{n-r}}{(q;q^{1/3})_{2n}(q^{1/3};q^{1/3})_{n-3r}(q;q)_r}=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+9n)/2}\right)$$

$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0, 1, 10$ (mod 11) (3.2.10)

3.3 Rogers-Ramanujan Type Identities Modulo 18:

Setting $a = 1, q^2$ successively in the transformations (2.2), (2.4), (2.6) and (2.8) respectively, we find the following identities of Rogers-Ramanujan Type:

$$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(2n^2+6n)/3}\,(q^2;q^2)_n}{(q^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_n}=\prod_{n=0}^{\infty}\frac{1}{1-q^n},$$

where $n \not\equiv 0, 2, 16$ (mod 18) (3.3.1)

$$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(2n^2+12n+18)/3}\,(q^2;q^2)_{n+1}}{(q^2;q^{2/3})_{2n+3}(q^{2/3};q^{2/3})_n}=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{9n^2-n} + q^6\sum_{n=-\infty}^{\infty}(-1)^n q^{9n^2+15n}\right)$$

$=\prod_{n=0}^{\infty}\frac{1}{1-q^n} + q^6\prod_{n=0}^{\infty}\frac{1}{1-q^n}$ (3.3.2)

where $n \not\equiv 0, 8, 10 (\text{mod } 18)$ and $n \not\equiv 0, 6, 12$ (mod 18)

$$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(2n^2+4n)/3}\,(q;q)_n}{(q^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_n}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+5n)}$$

$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$,

where $n \not\equiv 0, 4, 14$ (mod 18) (3.3.3)

$$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(2n^2+10n+12)/3}\,(q^2;q)_{n+1}}{(q^2;q^{2/3})_{2n+3}(q^{2/3};q^{2/3})_n}=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+13n)} + q^4\sum_{n=-\infty}^{\infty}(-1)^n q^{(9n^2+3n)}\right)$$

$=\prod_{n=0}^{\infty}\frac{1}{1-q^n} + q^4\prod_{n=0}^{\infty}\frac{1}{1-q^n}$ (3.3.4)

where $n \not\equiv 0, 4, 14 (\text{mod } 18)$ and $n \not\equiv 0, 6, 12$ (mod 18)

$$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(2n^2+2n)/3}\,(q^2;q^2)_n}{(q^{2/3};q^{2/3})_{2n+1}(q^{2/3};q^{2/3})_n}=\prod_{n=0}^{\infty}\frac{1}{1-q^n},$$

where $n \not\equiv 0, 6, 12$ (mod 18) (3.3.5)

$$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(2n^2+8n+6)/3}\,(q^2;q^2)_{n+1}}{(q^2;q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_n}=\prod_{n=0}^{\infty}\frac{1}{1-q^n} + q^2\prod_{n=0}^{\infty}\frac{1}{1-q^n}$$ (3.3.6)

where $n \not\equiv 0, 4, 14 (\text{mod } 18)$ and $n \not\equiv 0, 2, 16 (\text{mod } 18)$

$$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{2n^2/3}\,(aq^2;q^2)_{n-1}}{(aq^{2/3};q^{2/3})_{2n-1}(q^{2/3};q^{2/3})_n}=\prod_{n=0}^{\infty}\frac{1}{1-q^n},$$

where $n \not\equiv 0, 8, 10$ (mod 18) (3.3.7)

$$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\frac{q^{(2n^2+6n)/3}\,(q^2;q^2)_n}{(q^2;q^{2/3})_{2n}(q^{2/3};q^{2/3})_n}=\frac{1}{(q;q)_\infty}\left(\sum_{n=0}^{\infty}(-1)^n q^{(9n^2+9n)}(1-q^{(16n+16)})(1-q^{2n+2}\right)$$

$= 1 + \prod_{n=0}^{\infty}\frac{1}{1-q^n}$,

where $n \not\equiv 0, 2, 16$ (mod 18) (3.3.8)

3.4 Rogers-Ramanujan Type Identities Modulo 22:

Setting $a = 1, q^2$ successively in the transformations (2.10), (2.12), (2.14), (2.16) and (2.18) respectively, we find the following identities of Rogers-Ramanujan Type:

$$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2+6n+6r)/3}(q^2;q^2)_{n-r}}{(q^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+9n)}$$

$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$,

where $n \not\equiv 0, 2, 20$ (mod 22) (3.4.1)

$$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2+12n+12r+24)/3}(q^2;q^2)_{n-r+1}}{(q^2;q^{2/3})_{2n+3}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\prod_{n=0}^{\infty}\frac{1}{1-q^n} + q^8\prod_{n=0}^{\infty}\frac{1}{1-q^n}$$ (3.4.2)

where $n \not\equiv 0, 10, 12 (\text{mod } 22)$ and $n \not\equiv 0, 8, 14$ (mod 22)

$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2+6n+6r)/3}(q^2;q^2)_{n-r}\,(1+q^{2r+2})}{(q^{2/3};q^{2/3})_{2n+2}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\frac{1}{(q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+7n)}$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$,
where $n \not\equiv 0,4,18 \pmod{22}$ (3.4.3)

$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2+12n+12r+18)/3}(q^2;q^2)_{n-r+1}\,(1+q^{2r+4})}{(q^2;q^{2/3})_{2n+3}\,(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}$
$=\frac{1}{(q;q)_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+3n)}+q^6\sum_{n=-\infty}^{\infty}(-1)^n q^{(11n^2+17n)}\right)$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^6\prod_{n=0}^{\infty}\frac{1}{1-q^n}$ (3.4.4)
where $n \not\equiv 0, 8, 14 \pmod{22}$ and $n \not\equiv 0, 6, 16 \pmod{22}$

$$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2-6)/3}(q^2;q^2)_{n-r-1}\,(q^{2r}+q^{4r+2}-1)}{(q^{2/3};q^{2/3})_{2n-1}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}$$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$,
where $n \not\equiv 0,6,16 \pmod{22}$ (3.4.5)

$$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2+6n+6r)/3}(q^2;q^2)_{n-r-1}\,(q^{2r}+q^{4r+4}-1)}{(q^{2/3};q^{2/3})_{2n-1}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}$$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^4\prod_{n=0}^{\infty}\frac{1}{1-q^n}$ (3.4.6)
where $n \not\equiv 0, 6, 16 \pmod{22}$ and $n \not\equiv 0,4,18 \pmod{22}$

$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2+6r)/3}(q^2;q^2)_{n-r-1}}{(q^{2/3};q^{2/3})_{2n-1}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$,
where $n \not\equiv 0,8,14 \pmod{22}$ (3.4.7)

$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2+6n+12r)/3}(q^2;q^2)_{n-r}}{(q^2;q^{2/3})_{2n}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}+q^2\prod_{n=0}^{\infty}\frac{1}{1-q^n}$ (3.4.8)
where $n \not\equiv 0,4, 18 \pmod{22}$ and $n \not\equiv 0,2,20 \pmod{22}$

$\frac{(q^{2/3};q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{q^{(2n^2+6r^2)/3}(q^2;q^2)_{n-r-1}}{(q^{2/3};q^{2/3})_{2n-1}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}$
$=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$, where $n \not\equiv 0,10,12 \pmod{22}$ (3.4.9) *and*
$\frac{(q^2;q^{2/3})_\infty}{(q;q)_\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{a^{n+r-1}q^{(2n^2+6r^2+6n+6r-6)/3}(q^2;q^2)_{n-r}}{(q^2;q^{2/3})_{2n}(q^{2/3};q^{2/3})_{n-3r}(q^2;q^2)_r}=\prod_{n=0}^{\infty}\frac{1}{1-q^n}$,
where $n \not\equiv 0,2,20 \pmod{22}$ (3.4.10)

4. Conclusion

This paper was motivated by the work of Andrew V. Sills method employed in [1] where some analytical aspect has been considered to derive some more identities of Rogers-Ramanujan Type of modulo 9, 11, 18, and 22. Some other identities may also be found by more inspection on the values of the parameter a. Also there is a scope of obtaining more identities by incorporating some particular identities from the Slatter's famous list of 130 identities of Rogers-Ramanujan type.

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