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Particle Breakage, Dilatancy and Critical State in Crushable Granular Soils: A Review

Jiaqi Liang, Changming Wang*

College of Construction Engineering, Jilin University, Changchun 130021, Jilin, China
Correspondence Author

Abstract: ***Particle breakage is a pervasive micro-scale damage process in granular soils under high pressures, intense shearing, or complex stress paths, and in soils composed of weak or porous particles. By reshaping particle-size distribution (PSD), morphology, and contact networks, breakage alters compressibility, dilatancy, stiffness and strength evolution, and the location (and sometimes uniqueness) of critical and limiting states. Research has shifted from macroscopic inference toward mechanism-based "macro–micro coupling," enabled by single-particle crushing tests, high-resolution imaging, high-pressure consolidation and large-strain shear/ring-shear testing, DEM fracture modelling, and energy-partition analyses. This review synthesizes representative studies on: breakage quantification and PSD evolution laws; limiting compression curves and micromechanical explanations based on fractal crushing and minimum-particle control; energy dissipation and modified stress–dilatancy relations including breakage work; critical-state non-uniqueness and higher-dimensional state description; and constitutive/numerical modelling strategies from probabilistic crushing to thermodynamically consistent internal variables. Special soils (carbonate sands, volcanic soils, diatomaceous soils) are discussed as benchmarks for transferability. Research priorities are proposed for building a unified, physically based framework that is identifiable from experiments and usable in engineering prediction under monotonic, cyclic, and stress-reversal loading.***

Keywords: Particle breakage, Dilatancy, Critical state, Energy dissipation, Fractal crushing, Reference state curve, Constitutive model, DEM fracture, Carbonate sand, Volcanic soil, Diatomaceous soil**.**

1. Introduction

Particle breakage—splitting, chipping, abrasion, and comminution—occurs when contact forces exceed particle strength under high confining pressure, large shear strains, or non-proportional stress paths. In crushable sands and weak granular soils, breakage can dominate volumetric response (contraction/dilatancy), peak and residual strength, stiffness degradation, creep, and cyclic accumulation. These effects influence design and interpretation in pile installation and end bearing, deep foundations, rockfill dams, ballast and backfills, and other high-pressure geotechnical environments.

Modern studies combine particle-scale testing (single-particle crushing, imaging-based failure tracking) and statistical descriptions of strength dispersion with specimen-scale tests (high-pressure consolidation, large-strain shear/ring shear, cyclic and stress-reversal paths) that quantify PSD and morphology evolution. Energy-based interpretations and critical-state concepts provide physically consistent links between crushing, dilatancy, and limiting/critical states. In parallel, DEM and continuum constitutive models embed breakage into internal variables to reproduce stress – strain – volume responses across stress paths.

This review focuses on four questions: (1) how breakage is quantified and related to stress/strain/energy variables; (2) why limiting compression curves are near-linear in double-log space and how fractal crushing explains it; (3) how breakage dissipation modifies stress–dilatancy and affects critical-state interpretation; and (4) how models and simulations represent breakage–dilatancy–critical-state coupling, including lessons from special granular soils.

2. Fundamental Concepts and Index System

Quantifying breakage in a comparable way is the starting point. Nakata et al. [1] introduced the particle survival probability curve from single-particle crushing tests: breakage is treated as probabilistic due to strength dispersion, internal flaws, and shape-dependent stress concentrations. Comparing survival probabilities with estimates of particle loading inside specimens provides a scale-bridging interpretation and helps explain scatter, rate effects, and stress-path sensitivity in specimen tests.

At the PSD level, Zhang et al. [2] proposed an effective breakage probability and formulated PSD evolution as a Markov chain across discrete loading steps. The next PSD state depends on the current state through transition probabilities, enabling reconstruction of breakage kinetics from discrete PSD measurements. The framework highlights that "breakage state" is not determined by a single end-point index but evolves in a path-dependent manner, which is especially relevant when only intermittent PSD measurements are available in high-pressure or large-strain tests.

For high-pressure compression, Pestana and Whittle [3] developed a four-parameter elasto-plastic compression model emphasizing a transition from rearrangement-dominated deformation at low stresses to crushing-dominated deformation at high stresses. The limiting compression curve (LCC) captures convergence of specimens with different initial densities under high pressure. Its value is practical: it offers parameter identifiability with limited tests while reflecting the physical role of crushing in controlling compressibility over a wide stress range.

McDowell [4] justified the near-linear limiting compression relation in log e–log σ (or log e–log p) space by combining fractal PSD formation with minimum-particle control. If crushing generates a fractal distribution, void space scales with the volume of the smallest particles; if the minimum particle size evolves with stress through a size-effect-controlled strength limit, void ratio becomes a power function of stress. This provides a micromechanical foundation for empirical regularities and suggests that compression curves can encode information about crushing mechanisms.

When crushing is significant, state description must remain consistent across low and high pressures. Javanmardi et al. [5] proposed a reference state curve (RSC) to define soil state over wide pressure and density ranges. For crushable sands, the RSC offers a stable baseline for state parameters even when breakage shifts compression and critical-state loci, facilitating comparison across datasets and supporting calibration of models intended for broad stress ranges.

3. Advances in Experimental Techniques and Observation Methods

Single-particle crushing tests quantify strength dispersion and clarify failure definitions. Nakata et al. [1] compared quartz and feldspar particles, distinguishing an initial failure force (often corner/microcrack controlled) from a peak failure force (global splitting). Linking survival curves to marked-particle observations in triaxial specimens provided a route to connect micro-scale randomness with specimen-scale PSD changes.

High-resolution imaging reveals the temporal sequence of crushing. Wang and Coop [6] recorded single-particle crushing with a high-speed microscope and showed microcrack initiation, stable growth, and unstable coalescence. Mineralogy, initial defects, and particle shape can shift dominant modes between splitting and chipping/pulverization; these observations explain why different “crushing strength” definitions can lead to different model predictions and why breakage criteria must reflect failure mode.

Specimen-scale high-pressure compression and large-strain tests quantify coupled PSD–morphology evolution. Altuhaifi and Coop [7] combined high-pressure compression with microscopy and morphology indices, showing that gradation, density, and mineral strength control breakage mode. Uniform sands can split rapidly near yield, causing fast PSD refinement and packing improvement; well-graded sands often break progressively by corner damage and abrasion, with different size fractions contributing as stress increases. Importantly, roughness and particle shape evolve together with PSD, feeding back into dilatancy, friction mobilization, and fabric anisotropy.

Complex and cyclic paths highlight history dependence. In drained cyclic triaxial tests on Dogs Bay carbonate sand, López-Querol and Coop [8] observed densification and reduced dilatancy capacity with PSD refinement and contact-network reorganization. Altuhafi et al. [9] showed that stress reversal around displacement piles changes contact networks and crushing localization, producing distinct breakage and dilatancy even at the same final stress state. These findings warn against using monotonic critical-state parameters to predict cyclic or installation-induced behavior without accounting for crushing history.

Multi-physics and multi-scale observation are particularly valuable for special soils. Hoang et al. [10] used small-strain stiffness and electrical resistivity in sand–diatom mixtures to infer changes in skeleton formation and pore connectivity. Zhang et al. [11] coupled compression, permeability, and microstructural observation to show that internal pores/cavities in diatom particles strongly influence compressibility and hydraulic behavior. Together, these studies demonstrate that for porous or structured particles, PSD alone may be insufficient; internal porosity and morphology can control both crushing susceptibility and macroscopic response.

4. Breakage–Dilatancy Coupling from an Energy-Dissipation Perspective

Energy frameworks provide path variables with clear physical meaning. Ueng and Chen [12] extended Rowe-type stress–dilatancy by accounting for breakage energy consumption in drained triaxial shear. They proposed using breakage-energy dissipation rate and specific-surface-area growth rate to quantify breakage severity and its impact on dilatancy and strength.

Tarantino and Hyde [13] added a breakage dissipation term to Taylor’s work-dissipation equation and confirmed experimentally in direct shear that total dissipation can be partitioned into sliding-friction and breakage dissipation. This supports interpreting critical state as a steady dissipation partition, not only as stabilization of stress and volumetric change.

Because plastic work and breakage work are path integrals, they can distinguish stress paths that end at similar (p, q, e) yet generate different histories of crushing and dilatancy. This provides a natural basis for internal-variable evolution laws and complements higher-dimensional state descriptions when critical-state non-uniqueness is observed.

5. Compression Curves, Fractal Crushing, and Reference-State Descriptions

Limiting compression is a robust signature of crushable soils. Pestana and Whittle [3] interpreted convergence to an LCC as the result of crushing-dominated fabric evolution at high pressures. McDowell [4] complemented this with fractal crushing and minimum-particle control, providing a micromechanical rationale for log e–log σ linearity and implying that minimum particle size evolution is a key hidden variable governing compressibility.

Altuhaifi and Coop [7] emphasized that compression curves reflect coupled evolution of PSD, shape, and surface texture. Rapid splitting in uniform sands can increase packing efficiency quickly; progressive abrasion and chipping in well-graded sands yields more gradual densification. Interpreting compression response without microstructural evidence may therefore lead to misleading extrapolation across gradations, minerals, or stress ranges.

The RSC of Javanmardi et al. [5] provides a unified baseline for defining state parameters across wide pressures, improving interpretation when crushing shifts compression and critical-state loci and offering a practical coordinate system for broad-range constitutive modelling.

6. Shear Dilatancy and the Critical State: Non-Uniqueness, Path Dependence, and Ultimate Structure

A central issue is whether the critical state is uniquely defined by (p, q, e) in crushable sands. Tengattini et al. [14] showed that when crushing-induced contraction competes with rearrangement-induced dilatancy, (p, q, e) may be insufficient for a unique critical-state description. Their thermodynamically consistent framework introduces internal variables capturing this competition and provides a mechanism-based explanation for observed non-uniqueness.

Miao and Airey [15] found that compression- and shear-induced crushing in carbonate sand can follow different PSD/morphology paths, yet sufficiently large strain or stress may lead to an ultimate state with stabilized strength, volumetric response, and gradation. The result supports the idea that, despite path differences, terminal structures may exist, but the route taken and the associated dissipation partition can differ.

Cyclic loading accumulates crushing and shifts dilatancy potential. López-Querol and Coop [8] reported progressive PSD refinement, densification, and reduced dilatancy capacity in drained cyclic triaxial tests. This implies that cyclic stable states may deviate from monotonic critical-state trends and that history-dependent crushing should be represented explicitly when predicting cyclic deformation and stiffness/strength evolution.

Stress reversal and installation-like histories are particularly important. Altuhafi et al. [9] demonstrated that stress reversal can reconfigure contact networks and alter where crushing occurs, producing different breakage and dilatancy under the same final stress. This has direct implications for pile–soil interaction and for interpreting in-situ test data influenced by construction history.

7. Constitutive Modelling and Numerical Simulation: From Statistical Crushing to Mechanism-Consistent State Variables

Modelling strategies span probabilistic PSD evolution, micromechanical simulation, and continuum constitutive formulations. Nakata et al. [1] embedded strength dispersion via survival-probability curves and linked specimen-scale PSD change to particle-scale statistics, enabling interpretation of variability. Zhang et al. [2] modelled PSD evolution as a Markov chain, providing a probabilistic, path-dependent description that can be calibrated from discrete PSD snapshots.

DEM fracture modelling provides a mechanism-consistent route to capture breakage–fabric coupling. McDowell and Harireche [16] represented grains as bonded-sphere agglomerates and calibrated bond strength distributions so that simulated particles reproduce Weibull size effects and strength statistics. This makes crushing an emergent bond-failure process and enables investigation of how breakage redistributes force chains, changes coordination number, and modifies anisotropy and dilatancy.

Continuum models balance physics and practicality. Pestana and Whittle [3] captured wide-range compression with few parameters, supporting engineering use. Tengattini et al. [14] incorporated crushing–dilatancy competition in a thermodynamically consistent framework to reproduce critical-state trends while explaining non-uniqueness and state dependence in crushable sands.

Key limitations are parameter identifiability and data requirements. Markov-chain models need sufficiently dense PSD sampling to constrain transition probabilities; mechanism-rich critical-state models require combined datasets from compression, shear, and large-strain limiting tests to separate crushing and dilatancy contributions. RSC-based state references [5] and energy-type internal variables [8] may help reduce dimensionality and improve transferability, but calibration protocols must be standardized to ensure comparability across laboratories and materials.

8. Implications from Special Granular Soils for the Breakage–Dilatancy Mechanism

Special soils provide stringent tests for theory transfer. Carbonate sands can crush significantly at relatively low pressures; volcanic soils exhibit fragile, porous particles with time-dependent evolution; diatomaceous soils feature extreme morphology and internal porosity that modifies both friction and crushing.

For carbonate sands, López-Querol and Coop [8] and Miao and Airey [15] showed that low strength and internal porosity can accelerate PSD refinement and alter dilatancy, making critical/steady states more history sensitive. Tarantino and Hyde [13] quantified breakage dissipation, supporting explicit coupling between dissipation, dilatancy, and strength in carbonate-sand modelling.

Brandes and Nakayama [17] reported that Hawaiian volcanic soils show strong time dependence and creep, consistent with progressive microcrack growth and crushing in porous or glassy particles. This motivates rate/time-dependent breakage descriptions and careful separation of particle crushing from other time-dependent mechanisms such as viscous rearrangement.

In diatomaceous materials, Hoang et al. [10] linked diatom addition to changes in compressibility, stiffness, and resistivity via pore connectivity and skeleton formation. Zhang et al. [11] showed that diatom internal cavities increase void ratio and compressibility and affect permeability, while Zhang et al. [18] attributed high friction angles to micro/nanoscale interlocking and surface features. These results imply that transferable models should incorporate measurable descriptors of morphology and internal porosity, not solely PSD.

9. Future Research Directions

Priority directions include: (i) in-situ multi-scale observation (e.g., X-ray CT, image-based tracking) to link PSD/morphology/contact-network evolution to energy partitions and internal variables; (ii) unified cumulative-crushing laws for cyclic and stress-reversal paths by combining Markov PSD evolution [2] with energy measures [8,9] so that transition rates are driven by plastic/breakage work; (iii) minimal yet identifiable higher-dimensional state-variable sets guided by non-uniqueness results [14] and wide-range references [5]; and (iv) engineering validation and transferability for special soils using measurable structure descriptors (porosity, roughness, cavity ratio, shell stiffness) and benchmark problems such as pile–soil interaction and rockfill deformation.

10. Overall Synthesis and Conclusions

Breakage–dilatancy–critical-state coupling in crushable granular soils is inherently history dependent and multi-mechanism. Survival-probability approaches [1] and Markov-chain PSD evolution [2] describe randomness and path dependence beyond end-point indices. Limiting compression behavior is supported by fractal crushing and minimum-particle control [4], while parsimonious engineering models remain valuable [3]. Energy-partition frameworks connect crushing to stress–dilatancy and interpret critical/ultimate states as steady dissipation partitions. Non-unique critical states in crushable sands [14] motivate higher-dimensional state descriptions and benefit from wide-range baselines such as the RSC [5.1]. Special soils—carbonate sands, volcanic soils [17], and diatomaceous soi [10,11,18]—show that morphology and internal porosity can strongly modulate crushing and dilatancy, so transferable models should incorporate measurable structure descriptors. Future progress depends on tighter closure among multi-scale observations, energy variables, and identifiable state systems that remain simple enough for routine calibration and reliable enough for engineering design.

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