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Ultrasoft B-Open Sets and Ultrasoft B-Continuous Functions

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Abstract: In this paper, a new class of generalized hypersoft open sets in hypersoft topological spaces, called hypersoft b-open sets, is introduced and studied. The relationships among hypersoft α-open sets, hypersoft semi-open sets, hypersoft pre-open sets and hypersoft β-open sets are dealt with.We have also investigated the concepts of hypersoft b-open functions and hypersoft b-continuous functions.

Keywords: hypersoft b-open sets, hypersoft b-closed sets, hypersoft b-continuous functions.

1 Introduction :

Molodtsov [7] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz [10] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al. [12], Aygunoglu and Aygun [4] and Hussain et al are continued to study the properties of soft topological space. Weak forms of soft open sets were first studied by Chen [5]. He investigated soft semi-open sets in soft topological spaces and studied some properties of it.Arockiarani and Arokialancy [3] are defined soft β-open sets and continued to study weak forms of soft open sets in soft topological space. Later, Akdag and Ozkan [1] defined soft α-open (soft α-closed) sets.

In the present paper, we introduce some new concepts in hypersoft topological spaces such as hypersoft b-open sets, hypersoft b-closed sets, hypersoft b-interior, hypersoft b-closure, and hypersoft b-continuous functions.

Throughout the paper, the space X and Y stand for hypersoft topological spaces with $(X, \tau_X, \mathbb{E})$ and $(Y, \tau_Y, \mathbb{K})$ assumed unless otherwise stated. Moreover, throughout this paper, a hypersoft mapping $f : X \to Y$ stands for a mapping, where $f : (X, \tau, \mathbb{E}) \to (Y, \nu, \mathbb{K})$, $u : X \to Y$ and $p : \mathbb{E} \to \mathbb{K}$ are assumed mappings unless otherwise stated.

2 Preliminaries :

Definition 2.1: [7]

Let U be an initial universe, $P(U)$ be the power set of U, and $E_1, E_2, ..., E_n$ be the pairwise of disjoint sets of parameters. Let A_i ,$B_i \subseteq E_i$ for $i = 1, 2, ..., n$. A pair $(F, A_1 \times A_2 \times ... \times A_n)$ is called a hypersoft set over U, where F is a mapping given by $F : A_1 \times A_2 \times ... \times A_n \to P(U)$.

Simply, we write the symbol $\mathbb{E}$ for $E_1 \times E_2 \times ... \times E_n$, and for the subsets of $\mathbb{E}$: the symbols $\mathbb{A}$ for $A_1 \times A_2 \times ... \times A_n$, and $\mathbb{B}$ for $B_1 \times B_2 \times ... \times B_n$. Clearly, each element in $\mathbb{A}$,$\mathbb{B}$ and $\mathbb{E}$ is an n-tuple element.

We can represent a hypersoft set $(F, \mathbb{A})$ as an ordered pair,

$$(F, \mathbb{A}) = (\alpha, F(\alpha)) : \alpha \in \mathbb{A}.$$

Definition 2.2: [12]

A hypersoft set $(F, \mathbb{A})$ over U is said to be a relative null hypersoft set, denoted by $(\phi, \mathbb{A})$, if for all $\alpha \in \mathbb{A}$, $F(\alpha) = \phi$.

Definition 2.3: [12]

A hypersoft set $(F, \mathbb{A})$ over U is said to be a relative whole hypersoft set, denoted by $(\psi, \mathbb{A})$, if for all $\alpha \in \mathbb{A}$, $F(\alpha) = U$.

If $\mathbb{A} = \mathbb{E}$, then the $\mathbb{A}$-universal hypersoft set is called a universal hypersoft set, denoted by $\bar{X}$.

Definition 2.4: [12]

The union of two hypersoft sets of $(F, \mathbb{A})$ and $(G, \mathbb{B})$ over the common universe X is the hypersoft set $(H, \mathbb{C})$, where $\mathbb{C} = \mathbb{A}\tilde{\cup}\mathbb{B}$ and for all $e \in \mathbb{C}$,

$$H(e) = \begin{cases} \text{F(e)}, & \text{if } e \in \mathbb{A} - \mathbb{B} \\ \text{G(e)}, & \text{if } e \in \mathbb{B} - \mathbb{A} \\ \text{F(e)} \cap G(e), & \text{if } e \in \mathbb{A} \cap \mathbb{B}. \end{cases}$$

We write $(F, \mathbb{A})\tilde{\cup}(G, \mathbb{B}) = (H, \mathbb{C})$.

Definition 2.5: [12]

The intersection $(H, \mathbb{C})$ of two hypersoft sets of $(F, \mathbb{A})$ and $(G, \mathbb{B})$ over the common universe X,denoted $(F, \mathbb{A})\tilde{\cap}(G, \mathbb{B})$, is defined as $\mathbb{C} = \mathbb{A} \cap \mathbb{B}$, and $H(e) = F(e)\tilde{\cap}G(\mathbb{B})$ for all $e \in \mathbb{C}$.

Definition 2.6: [12]

Let $(F,\mathbb{A})$ and $(G,\mathbb{B})$ be two hypersoft sets over a common universe X,$(F,\mathbb{A})\tilde{\subset}(G,\mathbb{B})$, if $\mathbb{A}\subset\mathbb{B}$, and $H(e)=F(e)\subset G(e)$ for all $e\in\mathbb{A}$.

Definition 2.7: [9]

Let τ_H be the collection of hypersoft sets over U, then τ_H is said to be a hypersoft topology on U if
(1) (ϕ,$\mathbb{E}$), (ψ,$\mathbb{E}$) $\in\tau_H$,
(2) the intersection of any two hypersoft sets in $\tau_H\in\tau_H$,
(3) the union of any number of hypersoft sets in $\tau_H\in\tau_H$.
The triplet (U, τ_H,$\mathbb{E}$) is called a hypersoft topological space over U and the members of τ_H are said to be hypersoft open sets in U.

Definition 2.8: [9]

Let (U, τ_H,$\mathbb{E}$) be a hypersoft space over U. A hypersoft set $(\mathbb{F},\mathbb{E})$ over U is said to be a hypersoft closed set in U, if its complement $(F,\mathbb{E})^c$ belongs to τ_H.

3 Hypersoft b-open sets :

In this section we introduce hypersoft b-open sets in hypersoft topological spaces and study some of their properties.

Definition 3.1:

Let (U, τ_H,$\mathbb{E}$) be a hypersoft topological space and $(F,\mathbb{A})$ be a hypersoft set over U,

(i) Hypersoft b-closure of a hypersoft set $(F,\mathbb{A})$ in X is denoted by hbcl($(F,\mathbb{A})$) $=\tilde{\cap}\{(F,\mathbb{E})\tilde{\supset}(F,\mathbb{E}):(F,\mathbb{E})$ is a hypersoft b-closed set of $X\}$.

(ii) Hypersoft b-interior of a hypersoft set $(F,\mathbb{A})$ in X is denoted by hbint($(F,\mathbb{A})$) $=\tilde{\cup}\{(O,\mathbb{A})\tilde{\subset}(F,\mathbb{A}):(O,\mathbb{A})$ is a hypersoft b-open set of $X\}$.

Clearly hbcl($(F,\mathbb{A})$) is the smallest hypersoft b-closed set over X which contains $(F,\mathbb{A})$ and hbint$(F,\mathbb{A})$ is the largest hypersoft b-open set over X which is contained in $(F,\mathbb{A})$.

Definition 3.2:

A hypersoft set $(F,\mathbb{A})$ in a hypersoft topological space X is called

(i) hypersoft b-open set iff $(F,\mathbb{A})\tilde{\subset}int(cl(F,\mathbb{A})))$
$\tilde{\cup}cl(int((F,\mathbb{A})))$

(ii) hypersoft b-closed set iff $(F,\mathbb{A})\tilde{\supset}int(cl(F,\mathbb{A})))$
$\tilde{\cap}cl(int((F,\mathbb{A})))$.

Definition 3.3:

Let $(X,\tau,\mathbb{E})$ be a hypersoft topological space over X and $(F,\mathbb{A})$ be a hypersoft set over X.

(i) The hypersoft interior of $(F,\mathbb{A})$ is the hypersoft set int $((F,\mathbb{A}))=\tilde{\cup}\{(O,\mathbb{A})\tilde{\subset}(F,\mathbb{A}):(O,\mathbb{A})$ is hypersoft open of X };

(ii) The hypersoft closure of $(F,\mathbb{A})$ is the hypersoft set cl$((F,\mathbb{A}))=\tilde{\cap}\{(F,\mathbb{E})\tilde{\supset}(F,\mathbb{A}):(F,\mathbb{E})$ is hypersoft closed of X };

(iii) The hypersoft semi-interior of $(F,\mathbb{A})$ is the hypersoft set hypersoftint $((F,\mathbb{A}))=\tilde{\cup}\{(O,\mathbb{A})\tilde{\subset}(F,\mathbb{A}):(O,\mathbb{A})$ is hypersoft semi-open of X };

(iv) The hypersoft semi-closure of $(F,\mathbb{A})$ is the hypersoft set hypersoftcl $((F,\mathbb{A}))=\tilde{\cap}\{(F,\mathbb{E})\tilde{\subset}(F,\mathbb{A}):(F,\mathbb{E})$ is hypersoft semi-closed of X }.

Definition 3.4:

A hypersoft set $(F,\mathbb{A})$ in a hypersoft topological space X is called

(i) hypersoft regular open (hypersoft regular closed) set if $(F,\mathbb{A})=cl(int(F,\mathbb{A}))(int(cl(F,\mathbb{A}))=(F,\mathbb{A})$.

(ii) hypersoft α-open (hypersoft α-closed) set if $(F,\mathbb{A})\tilde{\subset}\ int(cl((F,\mathbb{A})))(cl(int(cl(F,\mathbb{A})))\tilde{\subset}(F,\mathbb{A})$.

(iii) hypersoft pre-open (hypersoft pre-closed) set if $(F,\mathbb{A})\tilde{\subset}int(cl(F,\mathbb{A}))(cl(int(F,\mathbb{A})))\tilde{\subset}(F,\mathbb{A}))$.

(iv) hypersoft semi-open (hypersoft semi-closed) set if $(F,\mathbb{A})\tilde{\subset}cl(int(F,\mathbb{A}))(int(cl(F,\mathbb{A})))\tilde{\subset}(F,\mathbb{A}))$.

(v) hypersoft b-open (hypersoft b-closed) set if $(F,\mathbb{A})\tilde{\subset}cl(int(cl(F,\mathbb{A})))(int(cl(int(F,\mathbb{A})))$
$\tilde{\subset}(F,\mathbb{A}))$.

Theorem 3.5:

Let $(F,\mathbb{A})$ be any hypersoft set in a hypersoft topological space X. Then,

(i) hbcl$(F,\mathbb{A})^c)=\tilde{X}-hbint((F,\mathbb{A}))$.

(ii) hbint$(F,\mathbb{A})^c)=\tilde{X}-hbcl((F,\mathbb{A}))$.

Proof:
(i) Let hypersoft b-open set $(O,\mathbb{A})\tilde{\subset}(F,\mathbb{A})$ and hypersoft b-closed set $(F,\mathbb{E})\tilde{\supset}(F,\mathbb{A})^c$.Then,hbint($(F,\mathbb{A})$) $=\tilde{\cup}\{(F,\mathbb{E})^c:(F,\mathbb{E})$ is a hypersoft b-closed set and $(F,\mathbb{E})\tilde{\supset}(F,\mathbb{A})^c\}=\tilde{X}-\tilde{\cap}\{(F,\mathbb{E}):(F,\mathbb{E})$ is a hypersoft b-closed set and $(F,\mathbb{E})\tilde{\supset}(F,\mathbb{A})^c\}=\tilde{X}-hbcl((F,\mathbb{A})^c)$. Therefore, hbcl$((F,\mathbb{A})^c)=\tilde{X}-hbint((F,\mathbb{A}))$.
(ii) Let $(O,\mathbb{A})$ be hypersoft b-open set.Then,for a hypersoft b-closed set $(O,\mathbb{A})\tilde{\supset}(F,\mathbb{A})$,$(O,\mathbb{A})\tilde{\subset}(F,\mathbb{A})^c$. hbcl$((F,\mathbb{A})))=\tilde{\cap}\{(O,\mathbb{A})^c:(O,\mathbb{A})$ is a hypersoft b-open set and $(O,\mathbb{A})\tilde{\subset}(F,\mathbb{A})^c\}=\tilde{X}-hbint((F,\mathbb{A})^c)$.
Therefore, hbint $((F,\mathbb{A})^c)=\tilde{X}-hbcl((F,\mathbb{A}))$.

Theorem 3.6:

For a hypersoft set $(F,\mathbb{A})$ in a hypersoft topological space U

(i) $(F,\mathbb{A})$ is a hypersoft b-open set iff $(F,\mathbb{A})^c$ is a hypersoft b-closed set.

(ii) $(F,\mathbb{A})$ is a hypersoft b-closed set iff $(F,\mathbb{A})^c$ is a hypersoft b-open set.

Proof: Obvious from the Definition 3.2.

Theorem 3.7:

In a hypersoft topological space X

(i) Every hypersoft pre-open set is hypersoft b-open set.

(ii) Every hypersoft semi-open set is hypersoft b-open set.

Proof:

(i) Let $(F, \mathbb{A})$ be a hypersoft pre-open set in a hypersoft topological space X.
Then, $(F, \mathbb{A}) \tilde{\subset} int(cl((F, \mathbb{A})))$ which implies $(F, \mathbb{A}) \tilde{\subset} int(cl((F, \mathbb{A}))) \tilde{\cup} int((F, \mathbb{A})) \tilde{\subset} int(cl((F, \mathbb{A}))) \tilde{\cup} cl(int((F, \mathbb{A})))$.
Thus, $(F, \mathbb{A})$ is hypersoft b-open set.

(ii) Let $(F, \mathbb{A})$ be a hypersoft semi-open set in a hypersoft topological space X.
Then, $(F, \mathbb{A}) \tilde{\subset} cl(int((F, \mathbb{A})))$ which implies $(F, \mathbb{A}) \tilde{\subset} cl(int((F, \mathbb{A}))) \tilde{\cup} int((F, \mathbb{A})) \tilde{\subset} cl(int((F, \mathbb{A}))) \tilde{\cup} int(cl((F, \mathbb{A})))$.
Thus $(F, \mathbb{A})$ is hypersoft b-open set.

Theorem 3.8:

In a hypersoft topological space X, every hypersoft b-open (hypersoft b-closed) set is hβ-open(hβ-closed) set.

Proof:
Let $(F, \mathbb{A})$ be a hypersoft b-open set in X.
Then,$(F, \mathbb{A}) \tilde{\subset} cl(int((F, \mathbb{A}))) \tilde{\cup} int(cl((F, \mathbb{A}))) \tilde{\subset} cl(int(cl((F, \mathbb{A})))) \tilde{\cup} int(cl((F, \mathbb{A}))) \tilde{\subset} cl(int(cl((F, \mathbb{A}))))$.
As a result$(F, \mathbb{A})$ is hβ-open set.

Theorem 3.9:

In a hypersoft topological space X

(i) An arbitrary union of hypersoft b-open sets is a hypersoft b-open set.

(ii) An arbitrary intersection of hypersoft b-closed sets is a hypersoft b-closed set

Proof:

(i) Let $\{(F, \mathbb{A})_\alpha\}$ be a collection of hypersoft b-open sets. Then, for each α, $(F, \mathbb{A})_\alpha \tilde{\subset} cl(int((F, \mathbb{A})_\alpha)) \tilde{\cup} int(cl((F, \mathbb{A})_\alpha))$.
Now, $\tilde{\cup}((F, \mathbb{A})_\alpha) \tilde{\subset} \tilde{\cup}[cl(int((F, \mathbb{A})_\alpha)) \tilde{\cup} cl(int((F, \mathbb{A}))] \tilde{\subset} [cl(int(\tilde{\cup}((F, \mathbb{A})_\alpha)))) \tilde{\cup} int(cl(\tilde{\cup}((F, \mathbb{A})_\alpha))))]$. Hence $((F, \mathbb{A})_\alpha)$ is a hypersoft b-open set.

(ii) Similarly by taking complements.

Theorem 3.10:

In a hypersoft topological space $X, (F, \mathbb{A})$ is hypersoft b-closed (hypersoft b-open) set if and only if $(F, \mathbb{A}) = hbcl(F, \mathbb{A})((F, \mathbb{A}) = hbint(F, \mathbb{A}))$.

Proof:
Suppose $(F, \mathbb{A}) = hbcl((F, \mathbb{A})) = \tilde{\cap}\{(F, \mathbb{E}) : (F, \mathbb{E})$ is a hypersoft b-closed set and $(F, \mathbb{E}) \tilde{\supset} (F, \mathbb{A})\}$ that implies, $(F, \mathbb{A}) \in \tilde{\cap}\{(F, \mathbb{E})$ is a hypersoft b-closed set and $(F, \mathbb{E}) \tilde{\supset} (F, \mathbb{A})\}$, that implies $(F, \mathbb{A})$ is hypersoft b-closed set.
Conversely, suppose $(F, \mathbb{A})$ is a hypersoft b-closed set in X.
We take $(F, \mathbb{A}) \tilde{\subset} (F, \mathbb{A})$ and $(F, \mathbb{A})$ is a hypersoft b-closed.
Therefore,
$(F, \mathbb{A}) \in \tilde{\cap}\{(F, \mathbb{E}) : (F, \mathbb{E})$ is a hypersoft b-closed set and $(F, \mathbb{E}) \tilde{\supset} (F, \mathbb{A})\}$.
$(F, \mathbb{A}) \tilde{\subset} (F, \mathbb{E})$ implies,$(F, \mathbb{A}) = \tilde{\cap}\{(F, \mathbb{E}) : (F, \mathbb{E})$ is a hypersoft b-closed set and $(F, \mathbb{E}) \tilde{\supset} (F, \mathbb{A})\} = hbcl((F, \mathbb{A}))$.
For $(F, \mathbb{A}) = hbint((F, \mathbb{A}))$ we apply hypersoft interiors.

Theorem 3.11:

In a hypersoft topological space X the following hold for hypersoft b-closure,

(i) hbcl(ϕ) $= \phi$.

(ii) hbint(ϕ) $= \phi$.

(iii) hbcl($F, \mathbb{A}$) is a hypersoft b-closed set in X.

(iv) hbcl(hbcl(F,$\mathbb{A}$))=hbcl((F,$\mathbb{A}$)).

Proof:
The Proof is obvious.

Theorem 3.12:

In a hypersoft topological space X the following relations hold;

(i) hbcl((F,$\mathbb{A}$))$\tilde{\cup}$(F,$\mathbb{B}$))$\tilde{\supset}$hbcl((F,$\mathbb{A}$))$\tilde{\cup}$hbcl((F,$\mathbb{B}$)).

(ii) hbcl((F,$\mathbb{A}$))$\tilde{\cap}$(F,$\mathbb{B}$))$\tilde{\subset}$hbcl((F,$\mathbb{A}$))$\tilde{\cap}$hbcl((F,$\mathbb{B}$)).

Proof:
(i) $(F, \mathbb{A}) \tilde{\subset} (F, \mathbb{A}) \tilde{\cup} (F, \mathbb{B})$ or $(F, \mathbb{B}) \tilde{\subset} (F, \mathbb{A}) \tilde{\cup} (F, \mathbb{B})$ that implies $hbcl((F, \mathbb{A})) \tilde{\subset} hbcl((F, \mathbb{A}))) \tilde{\cup} (F, \mathbb{B}))$ or $hbcl((F, \mathbb{B})) \tilde{\subset} hbcl((F, \mathbb{A}))) \tilde{\cup} (F, \mathbb{B}))$.
Thus, $hbcl((F, \mathbb{A}))) \tilde{\cup} (F, \mathbb{B})) \tilde{\supset} hbcl((F, \mathbb{A})) \tilde{\cup} hbcl((F, \mathbb{B}))$.
(ii) Similar to that of (i).

Theorem 3.13:

In a hypersoft topological space X the following relations hold;

(i) hbint((F,$\mathbb{A}$))$\tilde{\cup}$(F,$\mathbb{B}$))$\tilde{\supset}$hbint((F,$\mathbb{A}$))$\tilde{\cup}$hbint((F,$\mathbb{B}$)).

(ii) $hbint((F, \mathbb{A})) \tilde{\cap} (F, \mathbb{B})) \tilde{\subset} hbint((F, \mathbb{A})) \tilde{\cap} hbint((F, \mathbb{B}))$.

Proof:
(i) $(F, \mathbb{A}) \tilde{\subset} (F, \mathbb{A}) \tilde{\cup} (F, \mathbb{B})$ or $(F, \mathbb{B}) \tilde{\subset} (F, \mathbb{A}) \tilde{\cup} (F, \mathbb{B})$ that implies $hbint((F, \mathbb{A})) \tilde{\subset} hbint((F, \mathbb{A}))) \tilde{\cup} (F, \mathbb{B}))$ or $hbint((F, \mathbb{B})) \tilde{\subset} hbint((F, \mathbb{A}))) \tilde{\cup} (F, \mathbb{B}))$.
Thus, $hbint((F, \mathbb{A}))) \tilde{\cup} (F, \mathbb{B})) \tilde{\supset} hbint((F, \mathbb{A})) \tilde{\cup} hbint((F, \mathbb{B}))$.
(ii) Similar to that of (i).

Theorem 3.14:

Let $(F,\mathbb{A})$ be a hypersoft b-open set in a hypersoft topological space X.

(i) If $(F,\mathbb{A})$ be a hr-closed set then $(F,\mathbb{A})$ is a hp-open set.

(ii) If $(F,\mathbb{A})$ be a hr-open set then $(F,\mathbb{A})$ is a hypersoft-closed set.

Proof:
Since, (F,$\mathbb{A}$) is a hypersoft b-open set, $(F,\mathbb{A})\tilde{\subset}cl(int((F,\mathbb{A})))\tilde{\cup}int(cl((F,\mathbb{A})))$,
(i) Now let $(F,\mathbb{A})$ be hr-closed set.Therefore, $(F,\mathbb{A}) = cl(int((F,\mathbb{A})))$. Then, $(F,\mathbb{A})\tilde{\subset}(F,\mathbb{A})\tilde{\cup}int(cl((F,\mathbb{A})))$ that implies $(F,\mathbb{A})\tilde{\subset}cl(int((F,\mathbb{A})))$. Hence,(F,$\mathbb{A}$) is hypersoft r-open set.
(ii) Now let $(F,\mathbb{A})$ be hyper r-closed set.
Therefore, $(F,\mathbb{A}) = int(cl((F,\mathbb{A})))$.
Then, $(F,\mathbb{A})\tilde{\subset}(F,\mathbb{A})\tilde{\cup}cl(int((F,\mathbb{A})))$ that implies
$(F,\mathbb{A})\tilde{\subset}int(cl((F,\mathbb{A})))$. Thus,$(F,\mathbb{A})$ is hypersoft-open set.

4 Hypersoft b-continuity :

In this section, we introduce hypersoft b-continuous maps, hypersoft b-irresolute maps, hypersoft b-closed maps and hypersoft b-open maps and study some of their properties.

Definition 4.1:

Let $(X,\mathbb{E})$ and $(Y,\mathbb{K})$ be a hypersoft classes. Let $u : X \in Y$ and $p : \mathbb{E} \in \mathbb{K}$ be mappings. Then a mapping $f : (X,\mathbb{E}) \in (Y,\mathbb{K})$ is defined as: for a hypersoft set $(F,\mathbb{A})$ in $(X,\mathbb{E})$. $(f(F,\mathbb{A}),\mathbb{B}), \mathbb{B} = p(\mathbb{A}) \subseteq K$ is a hypersoft set in $(Y,\mathbb{K})$ given by
$f(F,\mathbb{A})(\beta) = u\left(\cup_{\alpha\in p^{-1}(\beta)\cap A}F(\alpha)\right)$ for $\beta \in K$. $(f(F,\mathbb{A}),\mathbb{B})$ is called hypersoft image of a hypersoft set $(F,\mathbb{A})$. If $\mathbb{B} = \mathbb{K}$, then we shall write $f(F,\mathbb{A}),\mathbb{K})$ as $f(F,\mathbb{A})$.

Definition 4.2:

Let $f : (X,\mathbb{E}) \to (Y,\mathbb{K})$ be a mapping from a hypersoft class $(X,\mathbb{E})$ to another hypersoft class $(Y,\mathbb{K})$,and $(G,\mathbb{C})$ a hypersoft set in hypersoft class $(Y,\mathbb{K})$,where C $\subseteq$ K: Let $u : X \to Y$ and $p : \mathbb{E} \to \mathbb{K}$ be mappings. Then $(f^{-1}(G,\mathbb{C}),D), D = p^{-1}(C)$, is a hypersoft set in the hypersoft classes $(X,\mathbb{E})$, defined as: $f^{-1}(G,\mathbb{C})(\alpha) = u^{-1}(G(p(\alpha)))$ for $\alpha \in D \subseteq E$. $(f^{-1}(G,\mathbb{C}),D))$ is called a hypersoft inverse image of $(G,\mathbb{C})$. Hereafter, we shall write $f^{-1}(G,\mathbb{C}),\mathbb{E}$ as $(f^{-1}(G,\mathbb{C}))$.

Definition 4.3:

A hypersoft mapping $f : X \to Y$ is said to be hypersoft b-continuous (briefly hypersoft b-continuous) if the inverse image of each hypersoft open set of Y is a hypersoft b-open set in X.

Definition 4.4:

A hypersoft mapping $f : X \to Y$ is called hypersoft b-continuous (resp. hypersoft α-continuous [1], hypersoft pre-continuous [1], hypersoft semi-continuous [2]) if the inverse image of each hypersoft open set in Y is hypersoft b-open (resp. hα-open, hp-open, hypersoft-open) set in X.

Definition 4.5:

A mapping $f : X \to Y$ is said to be hypersoft b-irresolute (briefly hypersoft b-irresolute) if $f^{-1}((F,\mathbb{K}))$ is hypersoft b-closed set in X, for every hypersoft b-closed set $(F,\mathbb{K})$ in Y.

Theorem 4.6:

Let $f : X \to Y$ be a mapping from a hypersoft space X to hypersoft space Y.Then the following statements are true;

(i) f is hypersoft b-continuous,

(ii) the inverse image of each hypersoft closed set in Y is hypersoft b-closed in X.

Proof:
(i) $\Rightarrow$ (ii): Let $(G,\mathbb{K})$ be a hypersoft closed set in Y.Then $(G,\mathbb{K})^c$ is hypersoft open set. Thus,$f^{-1}((G,\mathbb{K})^c) \in HbOS(X)$, i.e., $X - f^{-1}((G,\mathbb{K})) \in HbOS(X)$. Hence $f^{-1}((G,\mathbb{K}))$ is a hypersoft b-closed set in X.

(ii) $\Rightarrow$ (i): Let $(O,\mathbb{K})$ be a hypersoft open set in Y.Then $(O,\mathbb{K})^c$ is hypersoft closed set and by(ii) we have $f^{-1}((O,\mathbb{K})^c) \in HbCS(X)$, i.e., $X - f^{-1}((O,\mathbb{K})) \in HbCS(X)$.Hence $f^{-1}((O,\mathbb{K}))$ is a hypersoft b-open set in X. Therefore, f is a hypersoft b-continuous function.

Theorem 4.7:

Every hypersoft continuous function is hypersoft b-continuous function.
Proof:
Let $f : X \to Y$ be a hypersoft continuous function.
Let $(F,\mathbb{K})$ be a hypersoft open set in Y. Since f is hypersoft continuous, $f^{-1}((F,\mathbb{K}))$ is hypersoft open in X. And so $f^{-1}((F,\mathbb{K}))$ is hypersoft b-open set in X. Therefore, f is hypersoft b-continuous function.

Theorem 4.8:

A mapping $f : X \to Y$ is hypersoft b-irresolute mapping if and only if the inverse image of every hypersoft b-open set in Y is hypersoft b-open set in X.

Theorem 4.9:

Every hypersoft b-irresolute mapping is hypersoft b-continuous mapping.
Proof:
Let $f : X \to Y$ is hypersoft b-irresolute mapping. Let $(F,\mathbb{K})$ be a hypersoft closed set in Y, then $(F,\mathbb{K})$ is hypersoft b-closed set in Y. Since f is hypersoft b-irresolute mapping, $f^{-1}((F,\mathbb{K}))$ is a hypersoft b-closed set in X. Hence, f is hypersoft b-continuous mapping.

Theorem 4.10:

Let $f : (X, \tau, \mathbb{E}) \to (Y, \nu, \mathbb{K})$, $g : (Y, \nu, \mathbb{K}) \to (Z, \sigma, \mathbb{T})$ be two functions. Then

(i) $g \circ f : X \to Z$ hypersoft b-continuous, if f is hypersoft b-continuous and g is hypersoft continuous.

(ii) $g \circ f : X \to Z$ is hypersoft b-irresolute, if f and g is hypersoft b-irresolute functions.

(iii) $g \circ f : X \to Z$ is hypersoft b-continuous if f is hypersoft b-irresolute and g is hypersoft b-continuous.

Proof:

(i) Let $(H, \mathbb{T})$ be hypersoft closed set of Z. Since $g : Y \to Z$ is hypersoft continuous, by definition $g{-}1((H, \mathbb{T}))$ is hypersoft closed set of Y.Now $f : X \to Y$ is hypersoft b-continuous and $g{-}1((H, \mathbb{T}))$ is hypersoft closed set of Y, so by Definition 4.3, $f^{-1}(g{-}1((H, \mathbb{T}))) = (g \circ f)^{-1}((H, \mathbb{T}))$ is hypersoft b-closed in X. Hence $g \circ f : X \to Z$ hypersoft b-continuous.

(ii) Let $g : Y \to Z$ is hypersoft b-irresolute and let $(H, \mathbb{T})$ be hypersoft b-closed set of Z. Since g is hypersoft b-irresolute by Definition 4.5, $g^{-1}((H, \mathbb{T}))$ is hypersoft b-closed set of Y. Also $f : X \to Y$ is hypersoft b-irresolute, so $f^{-1}(g^{-1}((H, \mathbb{T}))) = (g \circ f)^{-1}((H, \mathbb{T}))$ is hypersoft b-closed. Thus, $g \circ f : X \to Z$ is hypersoft b-irresolute

(iii) Let $(H, \mathbb{T})$ be hypersoft b-closed set of Z. Since $g : Y \to Z$ is hypersoft b-continuous, $g^{-1}((H, \mathbb{T}))$ is hypersoft b-closed set of Y. Also $f : X \to Y$ is hypersoft b-irresolute, so every hypersoft b-closed set of Y is hypersoft b-closed in X. Therefore, $f^{-1}(g^{-1}((H, \mathbb{T}))) = (g \circ f)^{-1}((H, \mathbb{T}))$ is hypersoft b-closed set of X. Thus, $g \circ f : X \to Z$ is hypersoft b-continuous.

5 Conclusion :

In this paper, we introduce the concept of hypersoft b-open sets and hypersoft b-continuous functions in topological spaces and some of their properties are studied.

In the end, we hope that this paper is just a beginning of a new structure, it will be necessary to carry out more theoretical research to promote a general framework for the practical application.

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