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Application of Lopital's Rule in College Entrance Examination

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1. Definition

1.1 Type $\frac{0}{0}$ of Indeterminate Limit

If the function gf satisfies:

(i) $\lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to x_0} g(x) = 0$

B(ii) oth are x_0 differentiable $U^0(x)$ at some hollow $g'(x) \neq 0$ domain of the point, and:

(iii) $\lim\limits_{x \to x_0} \frac{f'(x)}{g'(x)} = A(A\ can\ be\ real\ or \pm \infty, \infty)$

Then

$$.\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)} = A$$

1.2 Type $\frac{\bullet}{\infty}$ of Indeterminate Limit

If the function gf satisfies:

B(i) oth x_0 are differentiable $U_+^0(x_0)$ in a certain $g'(x) \neq$ 0right domain, and;

; (ii) $\lim\limits_{x \to x_0^+} g(x) = \infty$

, (iii) $\lim\limits_{x \to x_0^+} \frac{f'(x)}{g'(x)} = A(Ac\ an\ be\ real\ or \pm \infty, \infty)$

Then

$$\lim_{x \to x_0^+} \frac{f(x)}{g(x)} = A$$

1.3 Other Types of Indefinite Limits

There are also types $0 \cdot \infty, 1^\infty, 0^0, \infty^0, \infty - \infty$ of infinitive limits and so on. After simple $\frac{0}{0}$ transformation $\frac{\infty}{\infty}$, they can generally be reduced to the limit of type or type.

2. Literature Review

2.1 Theoretical and Basic Applied Research

Li Zhongqing focuses on the final problem of the 2015 Fujian Science Exam, analyzing function inequality proofs to systematically elucidate the core role of L $\frac{0}{0}$ 'Hopital's Rule in solving limits and parameter ranges, particularly emphasizing its effectiveness in handling indeterminate forms [1]. He Changbin, starting from mathematical principles, provides a detailed introduction to the three basic forms of L 'Hopital's Rule and their applicable conditions, and combines it with basic problem types such as trigonometric function simplification and evaluation, demonstrating the advantages of constructing dual expressions in conjunction with L 'Hopital's Rule, providing theoretical support for high school teaching [2].

2.2 Problem Solving Strategies and Case Analysis Research

Li Hongguang used multiple real college entrance exam questions as examples, such as the 2011 National Paper and the 2015 Fujian Paper, to specifically demonstrate the practical application of L 'Hopital's Rule in separating parameters, finding extreme values, and solving inequality problems, emphasizing its efficiency compared to traditional case analysis [3]. Zhou Qixiang explored the flexible use of L 'Hopital's Rule in determining function extremum points and parameter ranges through complex cases like Question 21 of the 2017 National Paper III Science, pointing out that it should be supplemented with derivative definition methods for verification [4]. Xu Dongxue, starting from Question 21 of the 2011 National Paper Science, compared the differences in solving problems between the method of separating variables and L 'Hopital's Rule, proposing strategies to simplify complex problems through repeated differentiation and limit calculations, providing new ideas for high school students to tackle challenging final questions [5].

3. Three Examples

3.1 Question 18 of the New College Entrance Examination Mathematics Paper in 2024

known function $f(x) = ln\frac{x}{2-x} + ax + b(x-1)^3$

I(1) f, and $b = 0$, $f'(x) \geq 0$ find a the minimum value;

P(2) roof: the curve $y = f(x)$ is a central symmetric figure;

I(3) f and $f(x) > -2$only if$1 < x < 2$, the b range of values to be obtained.

S(1) lightly(2); slightly;

S(3) ince it $f(x) > -2$ holds and $1 < x < 2$ only when $x = 1$, $f(x) = -2$ it is a solution of

That $f(1) = -2$ is $a = -2$, so. $f(x) = ln\frac{x}{2-x} - 2x + b(x-1)^3$

Method direct method 1:

Then $1 < x < 2$, $ln\frac{x}{2-x} - 2x + b(x-1)^3 > -2$

Using the variable separation $b > \frac{2x-2-ln\frac{1+x}{1-x}}{(x-1)^3}$, we can know that.

a surname $g(x) = \frac{2x-2-ln\frac{1+u}{1-u}}{(x-1)^3}$,; $1 < x < 2$

Then $g'(x) = \frac{4-4x+\frac{2(x-1)}{x(2-x)}+3\,ln\frac{x}{2-x}}{(x-1)^4}$

a surname $h(x) = 4 - 4x + \frac{2(x-1)}{x(2-x)} + 3\,ln\frac{x}{2-x}$,; $h(1) = 0$

Then $h'(x) = \frac{4}{x^2(2-x)^2}[4x^3(1-x) + (2x+1-5x^2)]$

Then, $1 < x < 2$; $4x^3(1-x) + (2x+1-5x^2) < 0$

Therefore $h'(x) < 0$, $h(x)$ it $(1,2)$ is monotonically decreasing on the upper side;

For, $h(1) = 0$ launch $h(x) < 0$;

Therefore $g'(x) < 0$, $g(x)$ it $(1,2)$ is monotonically decreasing on the upper side.

According to the L 'Hopital's rule:

$$\lim_{x\to 1} g(x) = \lim_{x\to 1}\frac{2-\frac{2}{x(2-x)}}{3(x-1)^2} = \lim_{x\to 1}\frac{\frac{4(1-x)}{x^2(2-x)^2}}{6(x-1)} = \lim_{x\to 1}\frac{-2}{3x^2(2-x)^2} = -\frac{2}{3}$$

Therefore $b \geq -\frac{2}{3}$.

Method (The method of conversion)2:

The $u = x - 1$ rule $u \in (0,1)$ is $g(u) = f(u+1) + 2 = ln\frac{1+u}{1-u} - 2u + bu^3$,

Then $g(u) > 0$ and only $0 < u < 1$ then.

Using the variable separation $b > \frac{2u-ln\frac{1+u}{1-u}}{u^3}$, we can know that.

a surname $h(u) = \frac{2u-ln\frac{1+u}{1-u}}{u^3}$,; $0 < u < 1$

Then $h'(u) = \frac{1}{u^4}\left[\frac{-4u^3+6u}{u^2-1} + 2\,ln\frac{1+u}{1-u}\right]$

a surname $p(u) = \frac{-4u^3+6u}{u^2-1} + 2\,ln\frac{1+u}{1-u}$,, $p(0) = 0$; $0 < u < 1$

Then $p'(u) = -\frac{4u^4}{(u^2-1)^2}$ there are the latter;

At that $0 < u < 1$ time $p'(x) < 0$, $p(u)$ it $(0,1)$ was decreasing monotonically,

Therefore $p(0) = 0$; $p(u) < 0$

Therefore $h'(u) < 0$, $h(u)$ the $(0,1)$ function is monotonically decreasing on.

According to the L 'Hopital's rule:

$$;\lim_{x\to 1} f(x) = \lim_{u\to 0} h(x) = \lim_{u\to 0}\frac{2-\frac{2}{1-u^2}}{3u^2} = \lim_{u\to 0}\frac{-\frac{4u}{1-u}}{6u} = \lim_{u\to 0} -\frac{2}{3(1-u)} = -\frac{2}{3}$$

Therefore $b \geq -\frac{2}{3}$.

3.2 Question 21 of the National Science Paper 2024

known function $f(x) = (1 - ax)\,ln(1+x) - x$.

A(1) t that $a = -2$ time $f(x)$, we sought the extremum;

A(2) t that $x \geq 0$ time $f(x) \geq 0$, a the range of values to be obtained.

s(1) ummary

A(2) t that $x = 0$ time $f(0) = 0 \geq 0$, it was established;

then, $x > 0$; $(1 - ax)\,ln(1+x) - x > 0$

It can be known from $a \leq \frac{ln(1+x)-x}{x\,ln(1+x)}$ the use of variable separation that;

a surname $g(x) = \frac{ln(1+x)-x}{x\,ln(1+x)}$,

Then $g'(x) = \frac{\frac{x^2}{1+x}-ln^2(1+x)}{[x\,ln(1+x)]^2}$ there are the latter;

a surname $h(x) = \frac{x^2}{1+x} - ln^2(1+x)$, $h(0) = 0$

Then $h'(x) = \frac{x^2+2x-2(x+1)\,ln(1+x)}{(1+x)^2}$ there are the latter;

a surname $p(x) = x^2 + 2x - 2(x+1)\,ln(1+x)$,;$p(0) = 0$

Then $p'(x) = 2x - 2\,ln(1+x)$ there are the latter;

At that $x > 0$ time $p'(x) > 0$, $p(x)$ it $(0,+\infty)$ was increasing monotonically;

Therefore $p(0) = 0 p(x) > p(0) = 0$

Therefore $h'(x) > 0$, $h(x)$ it $(0,+\infty)$ is monotonically increasing;

Therefore $h(0) = 0$; $h(x) > h(0) = 0$

Therefore $g'(x) > 0$, $g(x)$ it $(0,+\infty)$ is monotonically increasing;

According to the L 'Hopital's rule:

$$;\lim_{x\to 0} g(x) = \lim_{x\to 0}\frac{\frac{1}{1+x}-1}{ln(1+x)+\frac{x}{1+x}} = \lim_{x\to 0}\frac{-\frac{1}{(1+x)^2}}{\frac{1}{x+1}+\frac{1}{(1+x)^2}} = \lim_{x\to 0}\frac{-1}{x+2} = -\frac{1}{2}$$

Therefore $a \leq -\frac{1}{2}$.

4. Problem Solving Steps

4.1 Problem Analysis and Variable Separation

The equations or inequalities containing multiple parameters are transformed into single parameter form by separating parameters. If the denominator is zero in the separation process, it needs to be analyzed separately to avoid missing critical situations.

4.2 Attempt of Loubid Rule

Determine whether the $g(x)$ separated function satisfies the conditions $g(x)$ of L 'Hopital's rule at the end point, and use L 'Hopital's rule to find the limit at the end point. If time is limited or derivation is blocked, you can still write down the process of L 'Hopital's rule, which may obtain step points.

4.3 Derivative Analysis and Simplification

For $g(x)$ the derivation $g'(x)$, $g'(x)$ we get that the parts (that do not affect the) sign, such as positive coefficients ($h(x)$ and square $h(x)$) terms, are normalized to keep the key factors unchanged. The key factors are kept unchanged. Repeat the above operation until the derivative function can be judged to be consistent.

4.4 Monotonicity and End Point Value Comprehensive Analysis

By determining the sign of the last primitive function on its domain through the final derivative, and combining this with the values of the original function (at the endpoints, we can analyze the sign of the second-to-last function on its domain). If the function is monotonically $g(x)$ increasing and has a non-negative value at the left endpoint, it can be concluded that the function is always greater than zero on its domain. Repeat this process until the monotonicity is fully established

4.5 Application of L 'Hopital's Rule

The numerator $g(x)$ and denominator are continuously differentiated until the denominator is not zero after being substituted into the end point, and then the limit can be obtained by substituting the function value.

4.6 Integration Conclusion of Parameter Ranges

Combined with the consistency of the separation of parameters and the obtained limit value, the range of values when the denominator is not zero is obtained; combined with the case of the denominator being zero, the range of values of the final parameters is obtained.

5. The Advantages of Solving Problems by Using the L'Hopital Rule

5.1 Advantages of Improving Problem Solving Efficiency

Conventional interval discussion requires the analysis of the properties of functions in different intervals within the domain one by one, which is tedious and easy to repeat. The L 'Hopital's rule has a fixed method in the process of use, which can save a lot of time and space.

5.2 Lower the Threshold of problem Solving for Students with Learning Difficulties

The traditional method strictly controls the range of values of independent variables, which requires strong logical coherence. Students with learning difficulties are prone to miss the critical situation or confuse the interval relationship, but the steps of using the L 'Hopital's rule are clear, so that students with learning difficulties can write down, so as to achieve the effect of improving their grades.

5.3 Help Students with Good Grades to Get Accurate Scores

Many students cannot achieve the effect of no repetition and no omission when solving derivative problems by traditional methods. Different methods are used for different intervals, which increases the difficulty of solving problems for students. However, the L 'Hopital's rule only needs to discuss the case where the denominator is zero, thus including all cases and reducing the point of losing points for high-achieving students, thereby maximizing the total score.

5.4 The Limit Idea of L 'Hopital's Rule

In the middle school stage, the theory of limit is only introduced at a shallow level, and there is no systematic application training. The idea of limit contained in l 'Hopital's rule can not only be applied to derivative problems, but also often appears in the final questions of selection and filling in the blanks.

5.5 Strengthen the Ability of Derivation

In the process of solving derivative problems by using L 'Hopital's rule, there are a lot of derivative calculations. Even if students cannot use L 'Hopital's rule in derivative problems in college entrance examination, they can find the derivative function faster and more accurately.

6. Recommendations

In the final problems of college entrance examination derivatives, when using the separation of variables method to handle parameter value ranges for always true problems, if the calculation of the (maximum $\frac{\infty}{\infty}\frac{0}{0}$ or) minimum values of the separated function is difficult, L 'Hopital's rule can serve as an effective tool -- to simplify calculations. In teaching, students should be guided to understand that its applicability is limited to indeterminate forms and complex limits, and through typical examples, demonstrate how to transform complex limits into differentiable forms. At the same time, emphasize that this method should complement traditional derivative analysis methods, such as first using the rule to quickly explore limit boundaries, then combining monotonicity and endpoint values to verify the rigor of the results, avoiding misuse due to neglecting the domain or non-differentiability points. Teachers should adhere to the principle of “looking at the big picture from a high perspective, starting from a low point, and implementing details meticulously”: starting with the limit concept from advanced mathematics, guide students to refine operations in steps like separating variables and parameter separation, strictly checking the legitimacy of each step. This not only enhances computational efficiency through standardized procedures but also helps students balance accuracy and logical completeness in solving problems within a limited time, ultimately achieving dual breakthroughs from tool application to mathematical literacy cultivation, providing new perspectives and strategies for tackling complex problems.

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