A Discussion on a New Concept of Open Sets in Simple Extended Topological Spaces

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Abstract: ***The purpose of this article is to introduce, and study the newly proposed concept of B^+C open set in simple extension topology. In this article we discuss some of the properties of B^+C open set, and obtain certain characterization and preserving theorems of B^+C-interior and B^+C-closure.***

Keywords: b^+open set,B^+C-open set

1. Introduction

The class of generalized open sets in a topologicalspaceis calledb-open sets was introduced by Andrijevic [1]. H. Z. Ibrahim [2] introduced the concept ofa BC-open set. In 1968, Velicko [10] introduced the concept of θ-open. Di Maio and Noiri [11] introduced the concept of semi-θ-open. R. H. Yunis[12] introduced the concept of properties of θ-semi open sets. In 1963, Levine [5] introduced the concept of simple extension of a topology$\tau(B) = \{(B \cap O) \cup O'/O, O' \in \tau$. F. Nirmala Irudayam [3] introduced the concept of b^+-open sets in extended topological spaces. The class of b^+-open sets is contained in the class of semi-pre$^+$open sets and contains all semi$^+$open and pre$^+$open sets. Joseph and Kwack [9] introduced the concept of θ-semi open sets using semi-open sets. It is well-known that a space X is called T_1if for, each pair of distinct points x, y of X there exists a pair of open sets, one containing x but not y and the other containing y but not x, as well as is T_1 if and only if for any point xϵX, the singleton set {x} is closed.In this article, (X,τ)stand for topological spaces with no separation axioms assumed unless otherwise stated.

2. Preliminaries

Definition 2.1
A subset A of a space X is called b^+open if$A \subseteq \mathrm{Int}(\mathrm{Cl}^+(A) \cup \mathrm{Cl}^+\mathrm{Int}(A))$. The family of all b^+open subsets of a simple extended topological space (X,τ^+) is denoted by $B^+O(X, \tau^+)$ or (Briefly.$B^+O(X)$).

Lemma: 2.2
For a subset A of a space (X,τ), the following conditions are equivalent:
1) A∈RO(X).
2) A∈ $\tau \cap$ SC(X).
3) A∈ $\alpha O(X) \cap$ SC(X).
4) A∈ $PO(X) \cap$ SC(X).

Definition: 2.3
A subset A of a space X is called ***θ*-semi-open** if for each xϵA,there exist a semi-open set G such that each xϵG ⊂ Cl(G) ⊂ A.

Definition: 2.4
A subset A of a space X is called **semi-*θ* -open** if for each xϵA,there exist a semi-open set G such that each xϵG ⊂ SCl(G) ⊂ A.

Definition: 2.5
A subset A of a space X is called ***θ* -open** if for each xϵA,there exist an open set G such that each xϵG ⊂ Cl(G) ⊂ A.

Theorem: 2.6
If X is s^{**}-normal, then $S\theta O(X) = \theta O(X) = \theta SO(X)$.

We recall that a topological space X is said to be extremely disconnected, if Cl(G) is open for every open set G of X.

Definition: 2.7
A space X is called locally indiscrete if every open subset of X is closed.

Theorem: 2.8
A space X is extremely disconnected if and only if $\delta O(X) = \theta SO(X)$.

Theorem: 2.9
A space X is extremely disconnected if and only if $RO(X) = RC(X)$.

3. On B^+C-Open set

This section introduces a new class of b^+-open sets namely B^+C-open sets in simple extended topological spaces and various properties of this novel set are studied.

Definition: 3.1
A subset A of a space (X, τ^+) is called B^+Copen, if for each $x \epsilon A \epsilon B^+O(X)$, there exists a closed set F such that$x \epsilon F \subset A$.
The family of all B^+Copen subsets is denoted by B^+CO(X) of a simple extended topological space (X, τ^+).

Theorem: 3.2
A subset A of a space X is B^+C open, if and only if A isb^+open and it is a union of closed sets in (X, τ^+).ie) A=∪ F_α.

That is A=$\cup F_\alpha$ where A is b^+open set, and F_α is closed sets in (X, τ^+) for eachα.

Proof:
The proof is obvious from the definition.

Remark: 3.3
Every B^+C open subset of a space (X, τ^+) is b^+open.

Remark: 3.4
The converse of the above remark need not be true, as shown inthe following example,

Example:3.5
Consider X= {1, 2, 3} with the topology
$\tau = \{\varphi, X, \{1\}, \{2\}, \{1,2\}\}$
τ^C={ φ, X, {2,3}, {1,3}, {3}}
B ={3}, $\tau^+(B) = \{(B \cap O) \cup O' / O. O' \epsilon \tau\}$

Then the family of closed sets are:
{ φ, X ,{1},{2},{2,3},{1,3},{2,3}}

Hence from the definitions we find the following families:
$B^+O(X)$= { φ, X ,{1},{2},{1,2},{1,3},{2,3}} and
$B^+CO(X)$= { φ, X ,{1,3},{2,3}}

Theorem:3.6
Let $\{A_\alpha : \alpha \epsilon \Delta\}$ be a collection of B^+Copen sets in a simple extended topological space (X, τ^+),then $\cup \{A_\alpha : \alpha \epsilon \Delta\}$ is B^+Copen.

Proof:
Let A_α be a B^+Copen set for eachα, then A_α isb^+open.
Hence $\cup \{A_\alpha : \alpha \epsilon \Delta\}$ is b^+open.
Let $x \epsilon \cup \{A_\alpha : \alpha \epsilon \Delta\}$, there exist $\alpha \epsilon \Delta$ such that$x \in A_\alpha$.
Since A_α is b^+open for each α, there exists a closed set F.
Such that$x \epsilon F \subset A_\alpha \subset \cup \{A_\alpha : \alpha \epsilon \Delta\}$.
So $x \epsilon F \subset \cup \{A_\alpha : \alpha \epsilon \Delta\}$.
Therefore, $\cup \{A_\alpha : \alpha \epsilon \Delta\}$ is B^+Copen set.

Theorem:3.7
If the family of allb^+open sets of a space X is a topology on X, then the family of B^+Copen sets is also a topology on X.

Proof:
Clearly $\varphi, X \epsilon B^+C$O(X) and by Theorem 3.6 the union of any family of B^+C open sets is B^+Copen.
To complete the proof it is enough to show that the finite intersection of B^+C open sets is B^+C open set.
Let A and B be two B^+Copen sets then A and B are b^+open sets.
Since B^+O(X) is a topology on X, so $A \cap B$ is b^+open.
Let $x \epsilon A \cap B$, then $x \epsilon A$and $x \epsilon B$, so there exists F and E such that $x \epsilon F \subset A$ and $x \epsilon E \subset B$ this implies that $x \epsilon F \cap E \subset A \cap B$.
Since intersection of closed sets is closed, $F \cap E$ is closed set.
Thus $A \cap B$ is B^+Copen set.
This completes the proof.

Theorem:3.8
The set A is B^+Copen in the simple extended topological space (X, τ^+), if and only if for each $x \epsilon A$, there exists a B^+Copen set B such that $x \epsilon B \subset A$.

Proof:
Assume that A is B^+Copen set.
Then for each $x \in A$, put A=B is B^+C open set containing X such that $x \in B \subset A$.
Conversely, suppose that for each $x \in A$, there exists a B^+C open set B such that $x \in B \subset A$
Thus $A = \cup B_x$ where $B_x \in B^+CO(X)$ for each x,therefore A is B^+C open set
In the following theorem, the family of b^+-open sets is identical to the family of B^+Copen sets.

Theorem:3.9
If a space (X, τ^+) isT_1-space, then the families $B^+O(X) = B^+CO(X)$.

Proof:
Let A be any subset of a space (X, τ^+)and$A \in B^+O(X)$.
If $A = \varphi, then\ A \in B^+CO(X)$.
If$A \neq \varphi$, then for each$x \in A$.
Since spaceX is T_1, then every singleton in (X, τ^+) is closed set.
Hence $x \in \{x\} \subset A$ and $A \epsilon B^+CO(X)$.
Thus$B^+O(X) \in B^+CO(X)$, but$B^+CO(X) \subset B^+O(X)$
Therefore$B^+O(X) = B^+CO(X)$.

Definition: 3.10
1) Let A be a subset of a simple extension topological space(X, τ^+) is called **θ^+- open set** if for each x∈ X,there exists an open set G such that x∈ G ⊂ (Cl^+(G)) ⊂ A.
2) Let A be a subset of a simple extension topological space(X, τ^+) is said to be **θ^+- semi open set** if there exists a θ^+- open set U of X such that U⊂ A ⊂ Cl^+(U).
3) Let A be a subset of a simple extension topological space (X, τ^+) called **δ^+- open set,** if for each x∈ A, there exists an open set G such that x∈ G ⊂ Int(Cl^+(G)) ⊂ A.
4) In simple extension topological space (X, τ^+) to be **s+**-normal** if and only if for every semi$^+$closed set F and every semi$^+$open set G containing F,there exist an open set H such that F⊂ H ⊂ Cl^+(H)) ⊂ G.
5) Let A be a subset of a topological space (X, τ^+) is **Regular$^+$-Open** if A=Int(Cl^+(A)).
6) Let A be a subset of a topological space (X, τ^+) is **Regular$^+$-Closed** if A=Cl^+(Int(A)).

Theorem:3.11
Every θ^+-semi open set of a space (X, τ^+) is B^+Copen set.

Proof:
Let A be a θ^+-semi open set in (X, τ^+), then for each $x \in A$, there exists a semiopen set G. Such that $x \in G \subset Cl^+(G) \subset A$, so $\cup \{x\} \in \cup G \cup Cl^+(G) \subset A$ for each $X \in A$implies that $A = \cup Cl^+(G)$, which is semi-open set and $A = \cup Cl^+(G)$ is a union of closed sets, by Theorem 3.2, A is B^+Copen set.

The following corollaries are the direct implications from the definition3.10-(5) and (6).

Corollary:3.12
1) Every θ^{+}-open set is $B^{+}C$open.
2) Every regular^{+}-closure is $B^{+}C$ open set.

Theorem:3.13
If a simple extended topological space (X, τ^{+}) is locally indiscrete, then $S^{+}O(X) \subset B^{+}CO(X)$.

Proof:
Let A be any subset of a space X
$A\epsilon S^{+}O(X)$, if A=φ then $A\epsilon B^{+}CO(X)$,
If$A \neq \varphi$,then$A \subset Cl^{+}Int(A)$.
Since X is locally indiscrete, then$Int(A)$ is closed
Hence$Int(A) \subset A$, this implies that for each$x\epsilon A$, $x\epsilon x(Int(A) \subset A)$.
Therefore, A is $B^{+}C$ open set.
Hence$S^{+}O(X) \subset B^{+}CO(X)$.

Remark: 3.14
Every open set in (X, τ^{+}) is semi^{+}-open, it follows that if a simple extended topological space (X, τ^{+}) is T_1 or locally indiscrete, then$\tau^{+} \subset B^{+}CO(X)$.

Theorem: 3.15
Let (X, τ^{+}) be a simple extended topological space, if X is regular, then $\tau^{+} \subset B^{+}CO(X)$.

Proof:
Let A be any subset of a simple extended topological space (X, τ^{+}).
Consider A be an open, if A=φ, then$A\epsilon B^{+}CO(X)$.
If$A \neq \varphi$, since X is regular, so for each$x \in A \subset X$, there exists an open set G such that
$x\epsilon G \subset Cl^{+}(G) \subset A$.
Thus we have $x\epsilon Cl^{+}(G) \subset A$.
Since $A\in \tau^{+}$ and hence$A\epsilon B^{+}O(X)$,
Therefore$\tau^{+} \subset B^{+}CO(X)$.

Theorem 3.16
Let(X, τ^{+}) be an extremely disconnected space and if$A \in \delta^{+}O(X)$, then$A\epsilon B^{+}CO(X)$.

Proof:
Let $A \in \delta^{+}O(X)$.
IfA=φ,then $A\epsilon B^{+}CO(X)$.
If $A \neq \varphi$,since a space X is extremely disconnected.
Then $\delta^{+}O(X) = \theta^{+}SO(X)$
Hence$A\epsilon\theta^{+}SO(X)$.
But$\theta^{+}SO(X) \subset B^{+}CO(X)$
Therefore, $A\epsilon B^{+}CO(X)$

Theorem: 3.17
Let (X, τ^{+}) be an extremelydisconnected space, if$A\epsilon R^{+}O(X)$, then$A\epsilon B^{+}CO(X)$.

Proof:
Theabove theorem can be proved easily using theorem 3.26, and the conditional that

$$R^{+}O(X) \subset \delta^{+}O(X)$$

Theorem:3.18
Let (X, τ^{+}) be an s^{+**}-normal space. If$A\epsilon S^{+}\theta O(X)$, then $A\epsilon B^{+}CO(X)$.

Proof:
Let $A\epsilon S^{+}\theta O(X)$.
IfA=φ,then $A\epsilon B^{+}CO(X)$.
If$A \neq \varphi$, since space X is s^{+**}-normal, $S^{+}\theta O(X) = \theta^{+}SO(X)$.
Hence $A\epsilon\theta S^{+}O(X)$.
But $\theta^{+}SO(X)\epsilon\, B^{+}CO(X)$.
Therefore, $A\epsilon B^{+}CO(X)$.

Theorem: 3.19
For any subset A of a simple extended topological space (X, τ^{+}) and$B^{+}O(X) = S^{+}\theta(X)$.
The following conditions are equivalent:
i) A is regular^{+}closed.
ii) A is closed and $B^{+}C$open.
iii) A is closed and b^{+}open.
iv) A is α-closed and b^{+}open.
v) A is pre-closed and b^{+}open.

Definition:3.20
A subset B of a space X is called $B^{+}C$closed, if X/B is $B^{+}C$ open. The family of all$B^{+}C$-closed subsets of a simple extended topological space(X, τ^{+}) is denoted by$B^{+}CC(X, \tau^{+})$ or (Briefly,$B^{+}CC(X)$).

Theorem:3.21
A subset B of a space X is $B^{+}C$-closed, if and only if B is a b^{+}-closed set, and it is an intersection of open sets.

Proof: Proof is straight forward.

Theorem:3.22
Let $\{b^{+}{}_{\alpha}: \alpha\epsilon\Delta\}$ be a collection of $B^{+}C$-closed sets in a topological space (X, τ^{+}).
Then $\cap \{b_{\alpha}: \alpha\epsilon\Delta\}$ is $B^{+}C$-closed.

Proof: The proof is analogous to theorem 3.6
The union of two $B^{+}C$-closed sets need not be $B^{+}C$ closed as is shown by the following counterexample,

Example: 3.23
In Example 3.5, the family of $B^{+}C$ closed subset of X is
$B^{+}CC(X) = \{\varphi, X, \{1\}, \{2\}\}$. Here $\{A\}\epsilon B^{+}CC(X)$ and
$\{B\}\ \epsilon B^{+}CC(X)$, but $\{1\} \cup \{2\} = \{1,2\} \notin B^{+}CC(X)$.
All of the following results are true by using complement.

Result: 3.24
If a space (X, τ^{+}) is T_1, then$B^{+}CC(X) = B^{+}C(X)$.

Result: 3.25
For any subset B of a space (X, τ^{+}). If $B\epsilon\theta^{+}SC(X)$, then $B\epsilon B^{+}CC(X)$.

Corollary: 3.26
Each θ^{+}-closed set is $B^{+}C$ closed.

Corollary: 3.27
Each regular$^+$open set is B^+C closed.

Remark: 3.28
If a simple extended topological space (X, τ^+) is locally indiscrete, then S^+C (X)$\subset B^+CC(X)$.

Corollary: 3.29
Let (X, τ^+) be a simple extended topological space, if X is regular or locally indiscrete, then the family of closed sets is a subset of the family of B^+C-closed sets.

Corollary: 3.30
Let (X, τ^+) be any extremelydisconnectedspace. If$B \in \delta^+C(X)$, then
$B \in B^+CC(X)$.

Corollary:3.31
Let (X, τ^+) be an extremely disconnected space. If$B \in R^+C(X)$, then$B \in B^+CC(X)$.

Corollary:3.32
Let (X, τ^+) be a s^{+**}-normal space. If $B \epsilon S^+\theta C(X)$, then$B \in B^+CC(X)$.

Theorem:3.33
For any subset B of a space(X, τ^+) andS^+C(X)=B^+C(X).
The following conditions are equivalent:
i) b^+ is regular$^+$open.
ii) b^+ is open and B^+C-closed.
iii) b^+ is open and b^+-closed.
iv) b^+ is α-open and b^+-closed.
v) b^+ is preopen and b^+-closed.

4. Some Properties of B+ C Open Sets

This section is devoted to the study of B^+C-neighborhood, B^+C-interior, B^+C -closure ofB^+Cderived set via the newly coined B^+C-open sets.

Definition:4.1
Let (X, τ^+) be a simple extended topological space and xϵX, then a subset N of x is said to be aB^+C-neighborhood of x, if there exists a B^+C-open set U in X such that x$\epsilon \cup\subset N$.

Theorem:4.2
In a simple extended topological space(X, τ^+), a subset A of X is B^+C-open, if and only if it is a B^+C-neighbourhood of each of its points.

Proof:
Let A$\subset X$ be a B^+C-open set,
since for everyxϵA, xϵA ⊂ A and A is B^+C-open,this shows that A is a B^+C- neighbourhood of each of its points.
Conversely, suppose that A is a B^+C-neighbourhood of each of its points.
Then for each xϵA, there exists $b^+x\epsilon B^+CO(X)$such that$b^+x \subset A$.
Then A=$\cup \{b_x : x\epsilon A\}$.
Since each b^+x is B^+C-open.
It follows that A is B^+C-open set.

Theorem: 4.3
For any two subsets A, B of a simple extended topological space (X, τ^+) and$A \subset B$, if A is a B^+Cneighbourhood of a pointxϵX, then B is also B^+C-neighbourhood of the same point x.

Proof:
Let A be a B^+C- neighbourhood ofxϵXand$A \subset B$ then by Definition 3.1.
There exists a B^+C-open set U such thatx$\epsilon \cup\subset A \subset B$.
This implies that B is also a B^+Cneighbourhood of x.

Remark: 4.4
Every B^+C neighborhood of a point is ab^+-neighbourhood,This follows from the fact that every B^+C-open set is b^+-open.

Definition: 4.5
Let A be a subset of a topological space(X, τ^+), a pointxϵX is said to be B^+C-interior point ofA, if there exist a B^+C-open set U such thatx$\epsilon \cup\subset A$. The set of allB^+C-interior points of A is called B^+Cinterior of A and is denoted byB^+CInt(A)

Some properties of the B^+C-interior of a set are investigated in the following theorem.

Theorem:4.6
For subsets A, B of a space X, the following statements hold,
i) B^+C Int(A)is the union of all B^+C-open sets which are contained in A.
ii)B^+C Int(A) is B^+C-open set in X.
iii)A is B^+C-open if and only if A = B^+C Int(A).
iv)B^+C Int(B^+C Int(A)) = B^+C Int(A).
v)B^+C Int(ϕ) = ϕ andB^+CInt(X) = X.
vi)B^+C Int(A) ⊂ A.
vii) IfA ⊂ B, thenB^+C Int(A) ⊂ B^+C Int(B).
viii) If A∩ B = ϕ, thenB^+C Int(A) ⊂ B^+C Int(B).
ix)B^+C Int(A) ∪ B^+C Int(B) ⊂ B^+C Int(A ∪ B).
x)B^+C Int(A ∩ B) ⊂ B^+C Int(A) ∩ B^+C Int(B).

Proof:
Let x∈ X and x∈ B^+C Int(A), then by Definition 4.5
There exists a B^+C-open set U such that x∈U⊂ $A \subset B$ implies that x∈U⊂ B.
Thus x∈ B^+C Int(B).
The other parts of the theorem can be proved easily.

Theorem:4.7.
For a subset A of a simple extended topological space(X, τ^+),then
B^+C Int(A) ⊂ b^+Int(A).

Proof:
This follows immediately since all B^+C-open set is b^+-open.

Definition: 4.8.
Let A be a subset of a space X. A point x∈ X is said to be B^+C-limit point of A, if for each B^+Copen set U containing x, U ∩ (A\{x}) ≠ φ. Then the set of all B^+C-limit points of

A is called a B^+C-derived set of A, and is denoted byB^+CD(A).

Theorem:4.9.
Let A be a subset of X, if for each closed set F of X containing x such that
$F \cap (A\backslash\{x\}) \neq \varphi$, then a point x$\in X$ is B^+C-limit point of A.

Proof:
Let U be anyB^+C-open set containing x.
Then for each x$\in U \in B^+O(X)$, there exists a closed set F such that x$\in F \subset U$.
By hypothesis, we have $F \cap (A\backslash\{x\}) \neq \varphi$.
Hence$U \cap (A\backslash\{x\}) \neq \varphi$.
Therefore, a pointx$\in X$ is B^+C-limit point of A.

Some properties of B^+C-derived set are stated in the following theorem,

Theorem:4.10.
Let A and B be subsets of a space X. Then we have the following properties:
i) $B^+CD(\varphi) = \varphi$
ii) IfX$\in B^+CD(A)$, thenx$\in B^+CD(A\backslash\{X\})$.
iii) If A$\subset$ B, then$B^+CD(A) \subset B^+CD(B)$.
iv) $B^+CD(A) \cup B^+CD(B) \subset B^+CD(A \cup B)$
v) $B^+CD(A\cap B) \subset B^+CD(A) \cap B^+CD(B)$
vi) $B^+CD(B^+CD(A))\backslash A\, B^+CD(A)$.
vii)$B^+CD(A\cup B^+CD(A) \subset (A \cup B^+CD(A))$

Proof: We only prove vi), vii), and the other resultscan be proved obviously.
vi)If x$\in B^+C\,D(B^+CD(A)\backslash A)$.

U is a B^+C-open set containing x.
$U \cap (B^+CD(A)\backslash\{x\}) \neq \varphi$.
Let y$\in U \cap (B^+CD(A)\backslash\{x\})$

Since y$\in B^+CD(A)$ andy$\in U, U \cap (A)\backslash\{y\}) \neq \varphi$.
Letz$\in U \cap (A)\backslash\{y\})$.
Then, z$\neq$ x for Z$\in$A andx$\notin$ A.
Hence $U \cap (A)\backslash\{x\}) \neq \varphi$.
Therefore, x$\in B^+CD(A)$.
vii) Let x$\in B^+CD(A \cup B^+CD(A))$.If x$\in$A the result is obvious.

Let x $\in B^+CD(A \cup B^+CD(A)\backslash A)$.

Then for B^+Copen set U containing x, $U \cap (A \cup B^+CD(A))\backslash\{x\}) \neq \varphi$.

Thus, $U \cap (A\backslash\{x\}) \neq \varphi$ or $U \cap (B^+CD(A)\backslash\{x\}) \neq \varphi$.
Now, it follows similarly from i) that $U \cap (A\backslash\{x\}) \neq \varphi$.
Hence, x$\in B^+CD(A)$.
Therefore, in any case$\left(B^+CD(A) \cup B^+CD(A)\right) \subset A \cup B^+CD(A)$.

Corollary:4.11.
For a subset A of a simple extended topological space (X, τ^+), then b^+D(A)$\subset B^+CD(A)$

Proof:
It is sufficient to recall that every B^+C-open set is b^+open.

Definition:4.12
For any subset A in a simple extended topological space (X, τ^+), the B^+C-closure of A, denoted by $B^+CCl^+(A)$, is defined by the intersection of all B^+C-closed sets containing A.

Theorem:4.13
A subset A of a simple extended topological space (X, τ^+) is B^+C-closed if and only if it contains the set of its B^+Climit points.

Proof:
Assume that A is B^+C-closed
If possible that x is a B^+C-limit point of A which belongs toX\A, then X\A is B^+C-open set containing the B^+C-limit point of A, therefore A$\cap X\backslash A \neq \varphi$.
which is a contradiction.
conversely, assume that A contains the set of its B^+Climit points.

For each x$\in X\backslash A$, there exists a B^+C-open set U containing X such that A$\cap U = \varphi$,
that is x$\in U \subset X\backslash A$ by Theorem 3.8, X\Ais B^+C -open set
Hence A is B^+C-closed set.

Theorem:4.14.
Let A be a subset of a space (X, τ^+), then$B^+CCl^+(A)$=A$B^+CD(A)$.

Proof:
Since $B^+CD(A) \subset B^+CCl^+(A)$ and A$\subset B^+CCl^+(A)$.
Then A$\cup B^+CD(A) \subset B^+CCl^+(A)$,on the other hand,
to prove that$B^+CCl^+(A) \subset A\, UB^+CD(A)$.
since$B^+CCl^+(A)$ is the smallest B^+C-closed set containing A,
so it is enough to prove that A$\cup B^+CD(A)$ is B^+C-closed.
Let x$\notin$A$\cup B^+CD(A)$.
This implies thatx$\notin$Aandx$\notin B^+CD(A)$.
Sincex$\notin B^+CD(A)$, there exists a B^+C-open set G(x) of x which contains no point of A other than x butx$\notin$A.
So G(x) contains no point of A, which implies G(x)$\subset X\backslash A$.
Again, G(x) is a B^+C-open set of each of its points.
But as G(x) does not contain any point of A, nopoint of G(x) can be a B^+C-limit point of A.
Therefore, no point of G(x) can belong to$B^+CD(A)$.
This implies that G(x)$\subset X\backslash B^+CD(A)$.
Hence, it follows that x$\in G(x) \subset X\backslash A \cap X\backslash B^+CD(A) \subset X(A \cup B^+CD(A))$
Therefore, $A \cup B^+CD(A)$ is B^+C-closed.
Hence$B^+CCl^+(A) \subset A \cup B^+CD(A)$.
Thus$B^+CCl^+(A) = A \cup B^+CD(A)$.

Corollary:4.15.
Let A be a set in a space (X, τ^+). A point x$\in$ U is in the B^+C-closure of A if and only if
A$\cap U \neq \varphi$, for every B^+C-open set U containing x.

Proof:
Let $x \notin B^+CCl^+(A)$.
Then $x \notin \cap F$, where F is B^+C-closed with $A \subset F$.
So $x \in X \backslash \cap F$ and $X \backslash \cap F$ is a B^+C-open set containing x.
Hence $(X \backslash \cap F) \cap A \subset (X \backslash \cap F) \cap (\cap F) = \varphi$.
Conversely, suppose that there exists a B^+C-open set containing x with $A \cap U = \varphi$.
Then $A \subset X \backslash U$ and $X \backslash U$ is a B^+C-closed.
Hence $x \notin B^+CCl^+(A)$.

Theorem:4.16
Let A be any subset of a space (X, τ^+). If $A \cap U \neq \varphi$ for every closed set F of X containing x, then the point x is in the B^+C-closure of A.

Proof:
Suppose that U be any B^+C-open set containing x, then by Definition 3.1, there exists a closed set F.
Such that $x \in F \subset U$, so by hypothesis, $A \cap F \neq \varphi$ implies $A \cap U \neq \varphi$ for every B^+C-open set U containing x.
Therefore $x \in B^+CCl^+(A)$.

Here we introduce some properties of B^+C-closure of the sets.

Theorem:4.17
For subsets A, B of a space (X, τ^+), the following statements are true.
1) The B^+C-closure of A is the intersection of all B^+Cclosed sets containing A.
2) $A \subset B^+CCl^+(A)$.
3) $B^+CCl^+(A)$ is B^+C-closed set in X.
4) A is B^+C-closed set if and only if $A = B^+CCl^+(A)$.
5) $B^+CCl^+(B^+CCl^+(A)) = B^+CCl^+(A)$.
6) $B^+CCl^+(\phi) = \phi$ and $B^+CCl^+(X) = X$.
7) If $A \subset B$, then $B^+CCl^+(A) \subset B^+CCl^+(B)$.
8) If $B^+CCl^+(A) \cap B^+CCl^+(B) = \phi$, then $AB = \phi$.
9) $B^+CCl^+(A) \cup B^+CCl^+(B) \subset B^+CCl^+(A \cup B)$.
10) $B^+CCl^+(A \cap B) \subset B^+CCl^+(A) \cap B^+CCl^+(B)$.

Proof: Obvious.

Theorem:4.18
For any subset A of a topological space (X, τ^+). The following statements are true.
1) $X \backslash B^+CCl^+(A) = B^+CInt(X \backslash A)$
2) $X \backslash B^+CInt(A) = B^+CCl^+(X \backslash A)$
3) $B^+CCl^+(A) = X \backslash B^+CInt(X \backslash A)$
4) $B^+CInt(A) = X \backslash B^+CCl^+(X \backslash A)$

Proof:
We only prove i), the other parts can be proved similarly.
For any point $x \in X$, $x \in X \backslash B^+CCl^+(A)$ implies that $x \notin B^+CCl^+(A)$.
Then for each $G \in B^+CO(X)$ containing x, $A \cap G = \varphi$.
Then $x \in G \subset X \backslash A$, thus $x \in B^+CInt(X \backslash A)$.

References

[1] D. Andrijevic, "On b-open sets", Mathmaticki. vesnik,59-64,48(1996).

[2] H.Z.Ibrahim, "Bc-open sets in topological spaces",Advances in Pure Math.,3,34-40,(2013).

[3] F.Nirmalairudayam, "Bc-open sets in extended topological spaces", IJAR 2(9),436-442,2016.

[4] A.S,Majid, "On some topological spaces by using b-open sets",M.S.C. Thesis University of AL-Qadissiya,college of mathematics and computer science,2011.

[5] N.Levine,"Semi-Open sets and Semi-Continuity in Topological Spaces,"American Mathematical Monthly,Vol.70.No.I.1963,pp.36-41.doi:10.2307/2312781.

[6] N.V.Velicko."H-closed Topological Spaces." American Mathematical society.Vol.38.No.2,1968,pp.103-118.

[7] R. H. Yunis, "Properties of θ-Semi-open sets."Zanco Journal Of Pure and applied sciences,Vol. 19, No.1,2007,pp. 116-122.

[8] N.K. Ahmed, "On Some Types of Separation Axioms,"M.Bc.Thesis,Salahaddin University ,Arbil,1990.

[9] J. E. Joseph and M. H. Kwack, "On S-Closed Spaces,"Bulletin of the American mathematical Society, Vol. 80, No.2,1980,pp.341-348.

[10] N.V. Velicko, "H-Closed Topological Spaces,"American Mathematical Society, Vol. 78, No. 2, 1968,pp. 103-118

[11] G. Di Maio and T.Noiri, "On s-Closed Spaces." Indian Journal of Pure and Applied Mathematics,Vol.18,No.3.1987,pp.226-233.

[12] R. H. Yunis, "Properties of θ-Semi-open sets."Zanco Journal Of Pure and applied sciences,Vol. 19, No.1,2007,pp. 116-122.

[13] M. H. stone, "Applications of the Theory of Boolean Rings to Topology,"Transactions of the American mathematical Society, Vol. 41, No.3,1937,pp.375-481. doi:10.1090/S0002-9947-1937-1501905-7.

[14] S. G.Crossley and S. K. Hildebrand, "semi-Closure,"Texas Journal of Science,Vol. 22,No.2-3,1971,pp.99-112.