DOI: 10.53469/jgebf.2026.08(03).09

An Alternative Approach to Estimating Brand Attraction Share: A Quantitative Comparison with the Colombo Morrison Model

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Generating a brand switching matrix (BSM) with Pi transformations as an alternate method of scaling to estimate attraction shares of brands and comparing them with the Colombo Morrison (C_M) model derivations

Abstract: ***Brand Studies have concentrated on choices made by buyers between brands and their impact on market shares of such brands. Most models measure purchase distributions by forming conditional probability tables or brand switching matrices (BSM). Largely such matrices consider the "first order" model condition of brand switching wherein probability of occurrence of any given outcome at a particular point in time depends on any previous outcomes of the buying process. Thus, market share of a brand is built by its attraction share i.e., the ability to of the brand to appeal buyers of other brands and its gravity share i.e., the ability of the brand to hold its current buyers from moving towards other brands. The process can be described as "inflow-outflow and steady state (I-O & S)" usage of buyers which is the cornerstone of "focus" and "gravity" share concept proposed by Colombo Morrison (C_M) in their paper published in Marketing Science,1989[1]. Their treatise considers focus as "appeal" or "attraction" and gravity as "retention" or "loyalty" stemming from the I-O & S state of users. The C_M model is based on deriving "attraction" and "loyalty" shares by directly using conditional probabilities (obtained in the BSM) by proportional draw method. This paper proposes to represent an alternate view of estimating "attraction" share through Pi transformation of the conventional BSM probabilities. All other estimations of the original C_M model and its theoretical objective remain unchanged. In conclusion, the paper compares the results obtained for "attraction" and "loyalty" shares under both scaling practices for similarity and validity.***

Keywords: Brand switching matrix, Pi transformation, loyals and switchers, attraction share

A brand switching model with Pi transformations

Alternate method of calculating "attraction share" used in the Colombo Morrison Model, Marketing Science, (1989)

C_M model overview

The model proposed by R.A. Colombo &D. G. Morrison is an extension of earlier work done on the "mover-stayer" class of models [2], where buyer choices exhibited loyalty and switching behaviour patterns. Most of the studies in this category typically constructed a brand switching matrix (BSM), which captures conditional probability distribution amongst buyer choices made between the last chosen brand and the current one. These "first-order" transitions are further employed to predict the terminal state of buyer acceptance (e.g., Markov chains) of the brand and eigen value-based market shares. First-order (F_O) model condition stress on "memory" effects with buyers who are cognizant of the last choice they made their next one. TheF_O condition assumes that buyers, while making choices between brands, look at some intrinsic derivable value in the purchase and can contrast between alternate brands given the differences of such values.

Relevance of the C_M model for marketers

Majority of the models on brand switching consider "loyal" buyers as monolithic. I.e., the buyers are entirely loyal since they purchased the same brand on two consecutive occasions thereby, by default, consider switchers also on similar basis. i.e., the brands only have "loyals" or "switchers". *The flaw in this assumption is that all repeat buyers are considered loyal*. However, sometimes a buyer may repeat purchase a given brand on two successive occasions due to non-availability of an alternate or a more preferred brand.

The C_M model's contribution stems from its proposition that "loyals" can be further split into hard core loyals and potential switchers. The split is an effective method in understanding buyers who while appearing to be *"loyal"* may only repeat purchase a brand due to compulsions such as non – availability of other brands or high transaction costs (e.g., travelling long distances to buy) incurred in buying them.

The C_M model's splitting of the "seemingly loyal" buyers indicate the marketer two important areas:

a) Which brands should we protect our buyer from buying on subsequent occasions?
b) Which brands should we target to increase our buyer share, given that there might be *latent switchers* in competing brands?

However, since the model works at an aggregate level with parsimonious data, it may not specify buyers individually who may remain with the brand or switch.

For purposes of clarity, discussed next is a brief quantitative overview of the original model.

1. Introduction to the model

The initial model proposed by Massy, Montgomery & Morrison 1970[2], assumes that buyers are either 'hard core loyal" or "potential switchers". The model assumes that a proportion α_1 of the previous buyers of the brand ***í***are perfectly loyal, i.e., the next brand they will buy will be brand ***í***.

The other previous buyers of brand ***í*** are perfectly non loyal potential switchers where the probability of their next purchase will be brand ***j*** given attraction of ***j*** is stated as $\Pi\, j$.

Thus, the probability of the previous buyer of brand ***í*** purchasing ***í*** is

$$P_{ii} = \alpha_1 + (1-\alpha_1)\, \Pi i \qquad (1)$$

And the probability of the previous buyer of brand ***í*** purchasing *j* is

$$P_{ij} = (1-\alpha_1)\, \Pi j \qquad (2)$$

Where:
α_1 represents the loyal buyers of the brand and thus $(1- \alpha_1)$ typifying the switchers

Estimation of Π
The attraction share of the brands is measured by the proportional conquest or "winning" share of the brand vs other brands in the consideration set [1].

Since the conditional loyalty share of the brand is known, we could estimate the proportion of the "loyals" and "switchers" in each brand given the movement of buyers in a transition matrix.

Consider the following probability transition matrix of farmers who bought various pesticides in the last season as compared to the current season.

Table 1: Brand switching / transition matrix used in this paper for illustration

		Current Purchase				
	Brand	A	B	C	D	E
Last Purchase	A	0.25	0.31	0.1	0.24	0.1
	B	0.1	0.21	0.35	0.25	0.09
	C	0.06	0.25	0.39	0.2	0.1
	D	0.1	0.15	0.25	0.35	0.15
	E	0.1	0.1	0.05	0.3	0.45

The matrix must be read as the "proportion of last user of the brand buying the next brand", where diagonal is retained proportion of buyers. i.e., 10% of Buyers of brand B purchased brand A in the current period (Row2 X Column1) and Brand A retained 25% of its last time users (Row1X Column1)

As mentioned, the C_M model derives the attraction share based on proportional draw of brands, i.e., First step is to estimate the proportional draw of Brand B over Brand A is 0.31/0.25 = ~ 1.24. (Table 2)

The second step is to consider the lowest column total or average for the brands and keep it as the starting point of 1 to derive a proportional draw of all brands from it. In case of the above values, the lowest column total is for Brand A. To estimate share of attraction of other brands from A, we keep Brand A's value as 1, (starting point) and proceed to derive values for other brands as times of Brand A. To illustrate how the shares appear, refer to below:

Table 2: Conversion into proportional attraction share of each brand

Brand	A	B	C	D	E
A					
B	1		3.5	2.5	0.9
C	1	4.17		3.33	1.67
D	1	1.5	2.5		1.5
E	1	1	0.5	3	

The total of the average of all columns is ~ 9.7.

If we were to estimate the attraction share of Brand B, it would be 2.22/9.7 = ~23% (A detailed note on the process is given in the original paper [1])

The same can be plugged in equation (1) above and loyalty& switcher shares can be derived from observed loyals. However, manual estimates are approximated for attraction share since marginal differences in stated numbers might creep and may not give exact results in the calibration. Nevertheless, the model is directionally sound for making decisions regarding which brands to "attack" to gain market share and which brands to "defend" market share from eroding to.

Postulate
Attraction shares work at two levels. Firstly, they draw buyers from other brands and secondly, they prevent existing buyers from migrating to rival brands. Weight of attraction, therefore, is the key factor in determining the switching matrix probabilities. *Increase in weight of attraction is possible by altering the scaling process in the BSM with Pi transformations*. The rest of the paper deals with the transformation matrix and the estimation of the new attraction and loyalty shares based on the data mentioned in Table 1.

Alternate Model of Attraction Share
A non-linear model to derive attraction share is proposed in this paper which can be later ratio scaled as the done in the C_M model.

Pi transformation of BSM probabilities
Brand's influence or attraction on buyers can also be visualized, geometrically speaking, as "area of attraction". We can imagine such influence in a "everywhere convex space", or a circle given equi-probability in brand choices for the buyer. Further, it can be suggested that the circumference of the brand's influence divided by its diameter is a constant Pi which can be used to derive a brand's "area of attraction" with the radius being "loyals" indicated in the BSM.

Estimation of brand's attraction area is illustrated below.
Let's consider Brand A's purchase proportion of manifest loyals from Table 1, above. The share stands at 0.25 (*Diagonal values are considered loyals*) with apparent total switchers being 0.75. Since, we posit the market being circular in space, the radius of attraction for brand A is 0.25 of its current share of "loyals" *which is also the share of its core loyals.*
Therefore,
Let *m* be the area of attraction of brand A:
Area of attraction or circle would be,
$A = \Pi r^2$
Thereby giving,
$m = 0.25^2 * \text{Pi}$
$= 0.20$
Likewise, area of attraction for other brands can be derived by using constant transformation subsequently yielding a new conversion matrix as under: (Data used is from the initial transition matrix given in Table 1)

Table 3: Area of attraction for individual brands using Pi transformation of attraction area probability

Force or Attraction Area

	A	B	C	D	E
A	0.20	0.30	0.03	0.18	0.03
B	0.03	0.14	0.38	0.20	0.03
C	0.01	0.20	0.48	0.13	0.03
D	0.03	0.07	0.20	0.38	0.07
E	0.03	0.03	0.01	0.28	0.64

Since the areas are mutually independent transformations, the row total does not add up to 1, as in case of the conditional probability matrix (See Table 1)

The above matrix is further used to derive the attraction share of the brands as follows:

Table 4: Matrix to be used for estimating brand attractiveness, like the C_M model stated earlier in the paper (Similar to matrix 2)

	A	B	C	D	E	
A						
B	1		12.3	6.25	0.81	
C	1	17.4		11.1	2.78	
D	1	2.25	6.25		2.25	
E	1	1	0.25	9		
Avg.	1	6.87	6.25	8.79	1.95	24.9
AS*	0.04	0.28	0.25	0.35	0.08	

AS Attraction Share*

The estimate of diagonal probability of loyal users of brand A

$\mathbf{P}_{ii} = \alpha_1 + (1-\alpha_1)\, \Pi i$

Ie.

$P_{AA} = \alpha_1 + (1-\alpha_1)\, 0.04$

=0.275(Observed was 0.25, Table 1, Row1 x Column 1)

(Discrepancies could arise in estimations since we round off numbers, &error adjustments. Also, excel solver used to derive precise results)

A full representation of the derived conditional matrix using constant Pi values is given below in Table 5.

Table 5: Projected or Derived conditional probability matrix given constant transformation value

	A	B	C	D	E
A	0.275	0.21	0.19	0.27	0.06
B	0.04	0.28	0.25	0.35	0.08
C	0.03	0.21	0.43	0.27	0.06
D	0.04	0.25	0.23	0.41	0.07
E	0.02	0.16	0.14	0.20	0.48

Correlation of the above table with the original brand switching matrix (Table 1) is 0.85 as compared to the C_M values which yields a correlation of 0.86.

Comparative values derived from both scaling methods

A comparative chart below indicates the values derived under both scaling methods. (In depth working of how the values are derived is beyond the scope of this paper as the treatise deals with the scaling of the probabilities and not final estimation or interpretation of loyals and switchers given in the C_M model). The reader is requested to see the original paper for the working [(1)].

Table 5: Table for comparison with values derived for "Loyals" and "Attraction" Share for both, C_M model of Proportional share by ratio scaling and PI converted ration scaling.

	Brands				
Ratio Scaling (Original C_M model)	A	B	C	D	E
Loyals	20%	0%	24%	15%	41%
Attraction	10%	25%	24%	29%	12%
PI scaling (Paper Proposed)					
Loyals	24%	0%	24%	8%	43%
Attraction	4%	28%	25%	35%	8%

Benefit of Pi transformation

The Pi transformation places more weightage on attraction share in deriving Loyalty shares of brands. For instance, in the above case, Brand D, seems to attract a larger proportion of users from other brands (See Table 1) as compared to say Brand E. The transformation, as seen from the above results, increases the weight of attraction for Brand D (29% to 35%) and reduces it for Brand E (12% to 8%).

The postulate for Pi transformation

As mentioned earlier in the paper, attraction shares cause buyers to choose between brands. The buyers also, despite attraction force of other brands, persist with some brands making them "loyal".

Thus, we postulate that "attraction shares" cause "loyalty shares" to occur and not vice versa. *Since attraction shares cause the final loyalty estimate, the Pi transformation values appear more logical in generating loyalty shares* as they increase attraction area thereby increasing brand's power in a proportional draw (Table 5, Brands D & E), as given above.

2. Further Work

The C_M model is a robust interpretation of the "mover-stayer" class of models indicating that loyals of any brand can be further split as hard core loyals and potential switchers. Areas of further study can be to evaluate validity of other transformation methods involving conditional probabilities in a conventional BSM. Additionally, it would be interesting to note if results alter after certain conditions of the model are relaxed. For instance, can loyalty be negative? *(Buyers buying the brand due to latent compulsions, but spreading negative buzz about it)*

References

[1] R. A. Colombo and D. G. Morrison (1989), "A brand switching model with implications for Marketing strategies", Marketing Science, issue 1, 89-99

[2] Massy, Montgomery, and Morrison (1970), Stochastic Models of Buying Behaviour, Cambridge, MA, The MIT Press, 464 p