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VIX Option Pricing under Hybrid Hawkes Jump-Diffusion with Stochastic Rates

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Abstract: ***This paper studies VIX option pricing when interest rates and volatility are random. It proposes an affine framework based on a mixed jump-diffusion model. This model uses a Vasicek random interest rate process and a jump component for random volatility. This helps capture interest rate risk, volatility risk, and jump clustering in financial markets. Under a consistent pricing framework, we build a combined system. This system includes Hawkes-type price jumps, volatility jumps, and random interest rates. We then derive the related generalized characteristic function. The pricing problem is solved using the Fourier Cosine Series Expansion (COS) method. Compared to traditional models, this extended model lowers the root mean square error in VIX option pricing. It performs especially well during periods of monetary policy changes and financial market stress. This research offers a new theoretical framework and empirical tools for pricing volatility derivatives in complex market settings.***

Keywords: VIX option pricing, Hawkes process, Stochastic interest rates, Stochastic volatility, COS method**.**

1. Introduction

VIX options are key derivatives for measuring market volatility, making their pricing an important research topic in financial engineering. Major crises like the 2008 financial crisis and the 2020 pandemic show that traditional pricing models fail to properly capture volatility movements during panic periods. Especially during major shifts in global monetary policy (like aggressive interest rate hikes in 2022-2023), the interaction between random interest rates and volatility risk becomes more prominent [1]. This creates two new challenges: On the one hand, changes in monetary policy frameworks (like the Fed's average inflation targeting) strengthen asymmetric links between rates and volatility [2] On the other hand, high-frequency trading environments amplify the self-excited spread of jump risks [3].

This paper builds the first unified affine model combining stochastic interest rates, stochastic volatility, and Hawkes jumps. It solves the joint pricing problem for volatility and price jumps under random interest rates. Current research has three main gaps: First, Hawkes process applications often assume constant interest rates [4], ignoring rate randomness. Second, traditional stochastic volatility models don't fully integrate volatility jumps with price jumps [5]. For example, while Eraker et al. [6] confirmed volatility jumps exist and Broadie et al. [7] built pricing frameworks, neither examined their interaction with interest rate dynamics.

Third, high-dimensional models cause "dimensionality problems": Fang et al.'s [8] COS method works well in two dimensions, but Zhang et al. [9] showed its convergence speed drops sharply in three or more dimensions. This leads to significant pricing errors during policy shifts, like Wang et al.'s [10] finding of abnormal rate sensitivity in VIX options on FOMC days.

This led researchers to adopt self-exciting point processes for jumps. For example, Merton [11] introduced compound Poisson processes but couldn't explain jump clustering;

Hawkes [12] solved this with self-exciting processes using contagion mechanisms; Bacry et al. [13] applied this to high-frequency pricing; Aït-Sahalia et al. [14] first used it for financial contagion modeling; Zhang et al. [15] explored its application to VIX futures. However, these models ignore how the random nature of interest rates affects jump intensity [16]. Research into the impact of random interest rates started with Ahn and others [17]. Bakshi and others [18] showed that it can cause pricing errors for long-term options of up to twenty percent. There is a policy-sensitive link between interest rates and volatility: Duffie and others [19] revealed their nonlinear relationship using affine jump-diffusion models. Stochastic volatility models have evolved from single-factor to multi-factor types. The Heston model struggles to capture the changing nature of volatility because it assumes constant variance [20]. Christoffersen et al. [21] improved their accuracy by adding the leverage effect. Erel et al. [22] showed that for every one percent increase in volatility jump size, the probability of a price jump increases by a factor of two point three. Also, for numerical calculations, the Fourier transform techniques face two major challenges. First, the FFT method by Carr et al. [23] works well in one dimension, but Tour et al. [24] showed that for three-dimensional models, the computational complexity grows exponentially. Second, the convergence speed of the COS method deteriorates sharply in more than three dimensions [25].

This paper makes two key innovations. The first is developing the first three-dimensional framework that combines the Vasicek interest rate model, the CIR volatility model, and Hawkes jumps. It uses an interest rate-jump coupling parameter to capture how policies amplify panic sentiment, overcoming limitations in the work by Zhu et al. [26] and Lian et al. [27]. The second is improving the COS method to boost efficiency for high-dimensional problems. Validated by Chau et al. [28], this makes calculations for three-dimensional models eight times faster, effectively solving the dimensionality issue. The rest of the paper is organized as follows: Section 2 describes the model assumptions. Section three presents the derivation of the characteristic function. Section four covers VIX option pricing. Section five provides the conclusions and future outlook.

2. Model Building

2.1 Basic Model and Assumptions

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\le t\le T}, Q)$ be a filtered probability space. Here Q is the risk-neutral probability measure, which E means the expectation under this measure Q. In this filtered probability space $(\Omega, \mathcal{F}, Q)$, we consider the following dynamics for the S&P 500 index, denoted by S_t:

$$\begin{aligned}\frac{dS_t}{S_{t-}} &= (R_t - m - \tilde{n}\lambda_t)dt + \sqrt{V_t}dW_{1,t} + (e^{Y_t} - 1)dN_t \\ dV_t &= \kappa_v(\theta_v - V_t)dt + \eta_v\sqrt{V_t}dW_{2,t} \\ dR_t &= \kappa_r(\theta_r - R_t)dt + \eta_r dW_{3,t}\end{aligned} \tag{2.1}$$

Here S_0 is the asset's initial price, m the continuous dividend yield, and $\tilde{n}$ the mean jump size. For parameters $\kappa_v, \theta_v, \eta_v$ are the mean-reversion speed, long-term average level, and volatility of volatility for the instantaneous variance $V_t, \kappa_r, \theta_r, \eta_r$ are the mean-reversion speed, long-term average level, and volatility for the instantaneous risk-free rate process R_t. We assume the Feller condition $2\kappa_v\theta_v \ge \eta_v^2$ holds to keep the stochastic volatility process V_t positive. $W_{1,t}, W_{2,t}, W_{3,t}, W_{4,t}$ They are all standard Brownian motions under the risk-neutral measure Q. Their correlations are constant $dW_{1,t}dW_{2,t} = \rho_{12}, dW_{1,t}dW_{3,t} = \rho_{13}$.

We assume the jump sizes $\{Y_t\}_{t\ge 1}$ are independent and identically distributed (i.i.d.) random variables. The term $e^{Y_t} - 1$ represents the percentage change caused by a price jump. In our model setup, each jump size Y_t follows a normal distribution $N(\mu_J, \sigma_J^2)$. Because the jump size is log-normally distributed, we can find the expected value of the original jump percentage:

$$\tilde{n} := \mathbb{E}(e^{Y_t} - 1) = exp\left(\mu_J + \frac{1}{2}\sigma_J^2\right) - 1 \tag{2.2}$$

$\{N_t\}_{t\ge 0}$ It is a Hawkes process with random intensity λ_t that changes over time. It satisfies the equation:

$$d\lambda_t = \kappa_\lambda(\theta_\lambda - \lambda_t)dt + \eta_\lambda dN_t \tag{2.3}$$

Here λ_0 is the initial value. The parameters $\kappa_\lambda, \theta_\lambda, \eta_\lambda$ are all positive constants. We assume $\kappa_\lambda > \eta_\lambda$ to ensure the intensity process is stationary. The full market model is then given by:

$$\begin{cases}\frac{dS_t}{S_{t-}} = (R_t - m - \tilde{n}\lambda_t)dt + \sqrt{V_t}dW_{1,t} + (e^{Y_t} - 1)dN_t \\ dV_t = \kappa_v(\theta_v - V_t)dt + \eta_v\sqrt{V_t}dW_{2,t} \\ dR_t = \kappa_r(\theta_r - R_t)dt + \eta_r dW_{3,t} \\ d\lambda_t = \kappa_\lambda(\theta_\lambda - \lambda_t)dt + \eta_\lambda dN_t\end{cases} \tag{2.4}$$

The model shows, when a jump occurs, the jump intensity process λ_t of the Hawkes process instantly increases by η_λ.

After this, the intensity decays exponentially at a rate κ_λ. This self-exciting property of the Hawkes process makes it well-suited for modeling jump clustering. We call this the SVJR model. This abbreviation will be used throughout the paper.

2.2 Calculating the Volatility Index

The Chicago Board Options Exchange (CBOE) significantly changed how it calculates the Volatility Index. Before September 2003, the VIX (now called VXO) was calculated using implied volatility from 8 at-the-money options on the S&P 100 index. These options had expiration dates closest to 30 calendar days. The current method was developed jointly by CBOE and Goldman Sachs. It uses a model-free approach to calculate implied volatility from S&P 500 index options.

We define $P(t, t+\tau) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} R_s ds}\right]$. This $P(t, t+\tau)$ is the price at the time t of a zero-coupon bond maturing at time $t+\tau$. According to [29], it has the form:

$$P(t, t+\tau) = exp\{E(\tau) - F(\tau)R_t\} \tag{2.5}$$

where

$$\begin{cases}E(\tau) = \left(\theta_r - \frac{\eta_r^2}{2\kappa_r^2}\right)(F(\tau) - \tau) - \frac{\eta_r^2 F^2(\tau)}{4\kappa_r} \\ F(\tau) = \frac{1 - e^{-\kappa_r\tau}}{\kappa_r}\end{cases} \tag{2.6}$$

The theoretical expression for the volatility index is:

$$\delta_t^2 = \frac{2}{\tau}\sum_i \frac{\Delta Z_i}{Z_i^2} P(t, t+\tau)\, Q_t(Z_i) - \frac{1}{\tau}\left(\frac{F_t}{Z_0}\right)^2 \tag{2.7}$$

Here $\delta_t = \frac{VIX_t}{100}$ is the VIX index divided by 100, $\tau = \frac{30}{365}$ represents the 30-calendar-day period in annualized terms, Z_i is the strike price of the i-th out-of-the-money (OTM) option on the S&P 500 index, $Q_t(Z_i)$ and is the The midpoint of the bid-ask spread for the option with strike Z_i at time t Z_0 is the first strike price below the forward price F_t at time, R_t which is the stochastic interest rate at time t.

The strike spacing ΔZ_i is calculated as $\Delta Z_i = \frac{Z_{i+1} - Z_{i-1}}{2}$ the current VIX calculation uses a model-free approach. It combines OTM call and put options on the S&P 500, where the weights exactly replicate a 30-day log contract. Using stochastic analysis, VIX squared can be expressed as the conditional expectation of the log contract under the risk-neutral measure (Derivations follow references [30]):

$$\delta_t^2 = \frac{2}{\tau}\mathbb{E}_t^{\mathbb{Q}}\left[\log\left(\frac{S_{t+\tau}}{F_t}\right)\right] \tag{2.8}$$

In the expression, $F_t = \frac{S_t}{P(t, t+\tau)} e^{-m\tau}$ the forward price of the underlying index at time t to maturity $t+\tau$ m is the continuous dividend yield of the underlying index. $\mathbb{E}_t^{\mathbb{Q}}$ Is the conditional expectation operator under the risk-neutral measure Q, information available at time t? Later, we will show that δ_t^2 it can be expressed as a closed-form expression using the model's state variables: the instantaneous variance. V_t The stochastic interest rate R_t and the jump intensity λ_t.

Proposition 2.1: Under the SVJR model (2.4), δ_t^2 at time t can be expressed as an affine function of the state variables V_t and λ_t:

$$\delta_t^2 = \varepsilon V_t + \alpha\lambda_t + \beta \tag{2.9}$$

The coefficients $\varepsilon, \alpha, \beta$ are given by:

$$\begin{cases} \varepsilon = \frac{1-e^{-\kappa_v \tau}}{\kappa_v \tau} \\ \alpha = 2(\tilde{n} - n_s + 1)\frac{1-e^{-\gamma\tau}}{(\kappa_\lambda - \eta_\lambda)\tau} \\ \beta = \theta_v(1-\varepsilon) + \frac{\kappa_\lambda \theta_\lambda}{\kappa_\lambda - \eta_\lambda}[2(\tilde{n} - n_s + 1) - \alpha] \end{cases} \quad (2.10)$$

Proof. At any time $t \le u \le s$, the dynamics of λ_t can be rewritten as:

$$d\lambda_t = \gamma(\xi - \lambda_t)dt + \eta_\lambda dM_t \quad (2.11)$$

Where $M_t = N_t - \int_0^t \lambda_u du$ is a local martingale, and $\gamma = \kappa_\lambda - \eta_\lambda, \xi = \frac{\kappa_\lambda \theta_\lambda}{\kappa_\lambda - \eta_\lambda}$. Define $\mu(t) = e^{\gamma t}$. Apply Itô's lemma to $\mu(t)\lambda_t$ and integrate both sides from t to $t+\tau$ We have:

$$\lambda_s = e^{-(s-t)\gamma}\lambda_t + \xi\left(1 - e^{-(s-t)\gamma}\right) + \eta_\lambda \int_t^s e^{(s-u)\gamma} dM_u \quad (2.12)$$

Taking conditional expectations on both sides gives:

$$\mathbb{E}_t^{\mathbb{Q}}(\lambda_s) = e^{-(s-t)\gamma}\lambda_t + \xi\left(1 - e^{-(s-t)\gamma}\right) \quad (2.13)$$

At any time, $t \le u \le s$ apply Itô's lemma to $e^{\kappa_v^t} V_t$ and integrate both sides from t to $t+\tau$, have:

$$V_s = e^{-\kappa_v(s-t)}V_t + \theta_v\left(1 - e^{-\kappa_v(s-t)}\right) + \eta_v \int_t^s e^{-\kappa_v(s-u)}\sqrt{V_u}dW_{2,u} \quad (2.14)$$

Taking conditional expectations on both sides gives:

$$\mathbb{E}_t^{\mathbb{Q}}(V_s) = e^{-\kappa_v(s-t)}V_t + \theta_v\left(1 - e^{-\kappa_v(s-t)}\right) \quad (2.15)$$

Similarly, the expression R_t is:

$$R_s = e^{-\kappa_r(s-t)}R_t + \theta_r\left(1 - e^{-\kappa_r(s-t)}\right) + \eta_r \int_t^s e^{-\kappa_r(s-u)}\sqrt{V_u}dW_{3,u} \quad (2.16)$$

Taking conditional expectations on both sides yields:

$$\mathbb{E}_t^{\mathbb{Q}}(R_s) = e^{-\kappa_r(s-t)}R_t + \theta_r\left(1 - e^{-\kappa_r(s-t)}\right) \quad (2.17)$$

Let $X_t \coloneqq \log S_t$. By Itô's lemma, the dynamics of the log-price M_t are:

$$d(\log(S_t)) = \left(R_t - m - \frac{1}{2}V_t + (\bar{n} - n_s + 1)\lambda_t\right)dt + \sqrt{V_t}dW_{1,t} + Y_t dM_t \quad (2.18)$$

where

$$\tilde{n} = \mathbb{E}_t^{\mathbb{Q}}[e^{Y_t} - 1] = n_s - 1, n_s = \mathbb{E}_t^{\mathbb{Q}}[e^{Y_t}], \bar{n} = \mathbb{E}_t^{\mathbb{Q}}[Y_t]$$

Integrating both sides from t to $t+\tau$:

$$\log\left(\frac{S_{t+\tau}}{S_t}\right) = \int_t^{t+\tau}\left(R_s - m - \frac{1}{2}V_s + (\bar{n} - n_s + 1)\lambda_s\right)ds + \int_t^{t+\tau}\sqrt{V_s}dW_{1,s} + \int_t^{t+\tau} Y_s ds \quad (2.19)$$

Taking conditional expectations:

$$\begin{aligned} &\mathbb{E}_t^{\mathbb{Q}}\left[\log\left(\frac{S_{t+\tau}}{S_t}\right)\right] \\ &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{t+\tau}\left(R_s - m - \frac{1}{2}V_s + (\bar{n} - n_s + 1)\lambda_s\right)ds\right] \\ &= \int_t^{t+\tau}\Big[e^{-\kappa_r(s-t)}R_t + \theta_r\left(1 - e^{-\kappa_r(s-t)}\right) - m \\ &\quad -\frac{1}{2}\left(e^{-\kappa_v(s-t)}V_t + \theta_v\left(1 - e^{-\kappa_v(s-t)}\right)\right) \\ &\quad +(\bar{n} - n_s + 1)\left(e^{-\gamma(s-t)}\lambda_t + \xi\left(1 - e^{-\gamma(s-t)}\right)\right)\Big]ds \end{aligned} \quad (2.20)$$

After computing and simplifying, we get:

$$\begin{aligned} &\mathbb{E}_t^{\mathbb{Q}}\left[\log\left(\frac{S_{t+\tau}}{S_t}\right)\right] \\ &= \left(\theta_r - m + (\bar{n} - n_s + 1)\xi - \frac{1}{2}V_t\right)\tau \\ &\quad + \frac{(R_t - \theta_r)(1 - e^{-\kappa_r \tau})}{\kappa_r} - \frac{(V_t - \theta_v)(1 - e^{-\kappa_v \tau})}{2\kappa_v} \\ &\quad +(\bar{n} - n_s + 1)\frac{(\lambda_t - \xi)(1 - e^{-\gamma\tau})}{\gamma} \end{aligned} \quad (2.21)$$

From the definition in (2.8), δ_t^2 it can be expressed as:

$$\delta_t^2 = \varepsilon V_t + \alpha\lambda_t + \beta$$

The coefficients $\varepsilon, \alpha, \beta$ are given by:

$$\varepsilon = \frac{1-e^{-\kappa_v \tau}}{\kappa_v \tau}$$

$$\alpha = 2(\tilde{n} - n_s + 1)\frac{1-e^{-\gamma\tau}}{\gamma\tau}$$

$$\beta = \theta_v(1-\varepsilon) + \xi[2(\tilde{n} - n_s + 1) - \alpha]$$

When we substitute $\gamma = \kappa_\lambda - \eta_\lambda$, $\xi = \frac{\kappa_\lambda \theta_\lambda}{\kappa_\lambda - \eta_\lambda}$ into these equations, we get the result we want.

During our research, we found that several classic pricing models are special cases of our general model, (1) When there are no jumps or jump clusters (i.e., parameters $n_s = 0, \sigma_s = 0, \lambda_t = 0$), our general model becomes the Heston model. The Heston model became very successful in stochastic volatility modeling mainly because it's easy to work with mathematically. Through careful verification, we confirm that the formula in [30] exactly matches our model in this case. This shows our model is logical and widely applicable. (2) Without jump clustering, when N_t a Poisson process with constant intensity λ, our model reduces to a stochastic volatility model with Poisson jumps. For this case, we derive the VIX-squared expression $\delta_t^2 = \varepsilon V_t + \alpha\lambda_t + \beta$ where $\varepsilon = \frac{1-e^{-\kappa_v \tau}}{\kappa_v \tau}$, $\alpha = \theta_v(1-\varepsilon) + 2(\bar{n} - n_s + 1)\lambda$.

This result relates closely to [26], but compared to their complex model with variance jumps, we use a simpler specification. We leave extensions with variance jumps for future research.

3. Characteristic Function

It's important to note that the SVJR model belongs to the class of affine jump-diffusion models. Given this, we use the method from Duffie et al. [19] to derive the model's characteristic function. Under the risk-neutral measure Q, the joint conditional characteristic function at time t is $\Psi(u_1, u_2, u_3; \nu, r, \lambda, \tau)$ then

$$\begin{aligned}&\Psi(u_1, u_2, u_3; \nu, r, \lambda, \tau)\\ &= \mathbb{E}_t^{\mathbb{Q}}\left[e^{iu_1V_T+iu_2R_T+iu_3\lambda_T}|\mathcal{F}_t\right]\\ &= \mathbb{E}_t^{\mathbb{Q}}\left[e^{iu_1V_T+iu_2R_T+iu_3\lambda_T}|V_T=\nu, R_T=R, \lambda_T=\lambda\right]\\ &= \mathbb{E}_t^{\mathbb{Q}}\left[e^{iu_1V_T+iu_2R_T+iu_3\lambda_T}\right]\end{aligned} \tag{3.1}$$

This $\Psi(u_1, u_2, u_3; \nu, r, \lambda, \tau)$ is the joint characteristic function of the random variables V_t, R_t, λ_t, given their values (ν, r, λ) at a time t. Here, $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ means the conditional expectation under the measure Q given the filtration $\{\mathcal{F}_t\}_{0\le t\le T}$. Also $\tau = T - t$ is the period $i = \sqrt{-1}$ is the imaginary unit, $u_1, u_2, u_3 \in \mathbb{C}$ are complex numbers. This joint characteristic function can be obtained by solving the following system of ordinary differential equations (ODEs), as shown in the next theorem.

Theorem 3.1: Under measure Q, the joint characteristic function $\Psi(u_1, u_2, u_3; V_t, R_t, \lambda_t, \tau)$ has this closed-form expression:

$$\begin{aligned}&\Psi(u_1, u_2, u_3; V_t, R_t, \lambda_t, \tau)\\ &= \exp\{iu_1\nu + iu_2 r + iu_3\lambda\\ &+A(u_1,u_2,u_3,\tau) + B(u_1,u_2,u_3,\tau)\nu\\ &+C(u_1,u_2,u_3,\tau)r + D(u_1,u_2,u_3,\tau)\lambda\}\end{aligned}$$

where

$$A(u_1,u_2,u_3,\tau) = \kappa_\nu\theta_\nu I_1 + \kappa_r\theta_r I_2 + \kappa_\lambda\theta_\lambda I_3 + \frac{1}{2}\eta_r^2 I_4$$

$$I_1 = \frac{2iu_1\kappa_\nu\tau}{a\kappa_\nu} - \frac{4\kappa_\nu}{a\kappa_\nu\eta_\nu^2}\log\left(\frac{2\kappa_\nu}{a+be^{-\kappa_\nu\tau}}\right)$$

$$I_2 = \frac{iu_2}{\kappa_r} - \left(\tau - \frac{1-e^{-\kappa_r\tau}}{\kappa_r}\right)$$

$$I_3 = \frac{iu_3}{\kappa_\lambda-\eta_\lambda} - \left(\tau - \frac{1-e^{-(\kappa_\lambda-\eta_\lambda)\tau}}{\kappa_\lambda-\eta_\lambda}\right)$$

$$I_4 = -\frac{u_3^2}{(\kappa_\lambda-\eta_\lambda)^2}\left(\tau - \frac{2\left(1-e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right)}{\kappa_\lambda-\eta_\lambda} + \frac{1-e^{-2(\kappa_\lambda-\eta_\lambda)\tau}}{2(\kappa_\lambda-\eta_\lambda)}\right)$$

$$B(u_1,\tau) = \frac{2iu_1(1-e^{-\kappa_\nu\tau})}{2\kappa_\nu - \eta_\nu^2 iu_1(1-e^{-\kappa_\nu\tau})}$$

$$C(u_2,\tau) = \frac{iu_2(1-e^{-\kappa_r\tau})}{\kappa_r}$$

$$D(u_3,\tau) = \frac{iu_3\left(1-e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right)}{\kappa_\lambda-\eta_\lambda}$$

$$a = 2\kappa_\nu - iu_1\eta_\nu^2$$

$$b = iu_1\eta_\nu^2$$

Proof. By the Feynman-Kac theorem, $\Psi(u_1, u_2, u_3; \nu, r, \lambda, \tau) \triangleq \Psi(\nu, r, \lambda, \tau)$ satisfies this partial differential equation (PDE):

$$\begin{aligned}&\frac{\partial\Psi}{\partial\tau} - \left(\kappa_\nu(\theta_\nu-\nu)\frac{\partial\Psi}{\partial\nu} + \kappa_r(\theta_r-r)\frac{\partial\Psi}{\partial r}\right.\\ &\left.+\kappa_\lambda(\theta_\lambda-\lambda)\frac{\partial\Psi}{\partial\lambda} + \frac{1}{2}\eta_\nu^2\nu\frac{\partial^2\Psi}{\partial\nu^2} + \frac{1}{2}\eta_r^2\frac{\partial^2\Psi}{\partial r^2}\right)\\ &-\lambda\mathbb{E}[\Psi(\nu,r,\lambda+\eta_\lambda,\tau) - \Psi(\nu,r,\lambda,\tau)] = 0\end{aligned} \tag{3.2}$$

Since the model has an affine structure [31], we assume the characteristic function takes this form:

$$\begin{aligned}&\Psi(u_1, u_2, u_3; V_t, R_t, \lambda_t, \tau)\\ &= \exp\{iu_1\nu + iu_2 r + iu_3\lambda\\ &+A(u_1,u_2,u_3,\tau) + B(u_1,u_2,u_3,\tau)\nu\\ &+C(u_1,u_2,u_3,\tau)r + D(u_1,u_2,u_3,\tau)\lambda\}\end{aligned} \tag{3.3}$$

With terminal conditions: $\Psi(u_1,u_2,u_3,0) = e^{iu_1\nu+iu_2r+iu_3\lambda}$ $A(u_1,u_2,u_3,0) = 0$, $B(u_1,u_2,u_3,0) = 0$, $C(u_1,u_2,u_3,0) = 0$, $D(u_1,u_2,u_3,0) = 0$.

Substituting this into (3.2) gives:

$$\begin{aligned}&\frac{\partial A}{\partial\tau} + \frac{\partial B}{\partial\tau}\nu + \frac{\partial C}{\partial\tau}r + \frac{\partial D}{\partial\tau}\lambda\\ &= \kappa_\nu(\theta_\nu-\nu)B + \kappa_r(\theta_r-r)C + \kappa_\lambda(\theta_\lambda-\lambda)D\\ &+\frac{1}{2}\eta_\nu^2\nu B^2 + \frac{1}{2}\eta_r^2 C^2 + \lambda(e^{D\eta_\lambda}-1)\end{aligned} \tag{3.4}$$

Substituting (3.3) into (3.2) and using (3.4), we find the unknown coefficient functions $A(u_1,u_2,u_3,\tau)$, $B(u_1,u_2,u_3,\tau)$, $C(u_1,u_2,u_3,\tau)$, and $D(u_1,u_2,u_3,\tau)$ satisfy these ordinary differential equations (ODEs):

$$\begin{cases}\frac{\partial A}{\partial\tau} = \kappa_\nu\theta_\nu B + \kappa_r\theta_r C + \kappa_\lambda\theta_\lambda D + \frac{1}{2}\eta_r^2 C^2,\\ A(u_1,u_2,u_3,0) = 0.\end{cases} \tag{3.5}$$

$$\begin{cases}\frac{\partial B}{\partial\tau} = \frac{1}{2}\eta_\nu^2 B^2 - \kappa_\nu B + iu_1,\\ B(u_1,u_2,u_3,0) = 0.\end{cases} \tag{3.6}$$

$$\begin{cases}\frac{\partial C}{\partial\tau} = -\kappa_\nu C + iu_2,\\ C(u_1,u_2,u_3,0) = 0.\end{cases} \tag{3.7}$$

$$\begin{cases}\frac{\partial D}{\partial\tau} = -\kappa_\lambda D + e^{D\eta_\lambda} - 1 + iu_3,\\ D(u_1,u_2,u_3,0) = 0.\end{cases} \tag{3.8}$$

Where Equation (3.6) is a constant-coefficient Riccati equation B, Equation (3.7) is a first-order linear ODE for C. We can solve these equations to get:

$$B(u_1,\tau) = \frac{2iu_1(1-e^{-\kappa_\nu\tau})}{2\kappa_\nu - \eta_\nu^2 iu_1(1-e^{-\kappa_\nu\tau})} \tag{3.9}$$

$$C(u_2,\tau) = \frac{iu_2(1-e^{-\kappa_r\tau})}{\kappa_r} \tag{3.10}$$

Usually, A and D can't be written in a simple closed form. But when the Hawkes process becomes a Poisson process, we get explicit solutions that match [26]. We'll solve the ODEs using the fourth-order Runge-Kutta method. Here's how:

Solving equation (3.8) with Runge-Kutta is slow because it needs step-by-step initial values. To make it faster, we approximate $e^{D\eta_\lambda}$ using its first-order Taylor expansion $e^{D\eta_\lambda} \approx 1 + D\eta_\lambda$. This turns (3.8) into a linear ODE:

$$\frac{\partial D}{\partial\tau} = (-\kappa_\lambda + \eta_\lambda)D + iu_3$$

After simple calculations, we get:

$$D(u_3,\tau) = \frac{iu_3\left(1-e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right)}{\kappa_\lambda-\eta_\lambda} \tag{3.11}$$

Plug (3.9), (3.10), and (3.11) into (3.5) and integrate:

$$A(u_1,u_2,u_3,\tau) = \kappa_\nu\theta_\nu I_1 + \kappa_r\theta_r I_2 + \kappa_\lambda\theta_\lambda I_3 + \frac{1}{2}\eta_r^2 I_4 \tag{3.12}$$

where

$$I_1 = \frac{2iu_1\kappa_V\tau}{a\kappa_V} - \frac{4\kappa_V}{a\kappa_V\eta_V^2}\log\left(\frac{2\kappa_V}{a+be^{-\kappa_V\tau}}\right)$$

$$I_2 = \frac{iu_2}{\kappa_r} - \left(\tau - \frac{1-e^{-\kappa_r\tau}}{\kappa_r}\right)$$

$$I_3 = \frac{iu_3}{\kappa_\lambda-\eta_\lambda} - \left(\tau - \frac{1-e^{-(\kappa_\lambda-\eta_\lambda)\tau}}{\kappa_\lambda-\eta_\lambda}\right)$$

$$I_4 = -\frac{u_3^2}{(\kappa_\lambda-\eta_\lambda)^2}\left(\tau - \frac{2\left(1-e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right)}{\kappa_\lambda-\eta_\lambda} + \frac{1-e^{-2(\kappa_\lambda-\eta_\lambda)\tau}}{2(\kappa_\lambda-\eta_\lambda)}\right)$$

$$a = 2\kappa_V - iu_1\eta_V^2$$

$$b = iu_1\eta_V^2$$

The solutions $A(u_1,\tau)$, $B(u_1,\tau)$, $C(u_2,\tau)$, $D(u_3,\tau)$ follow similar methods and then obtain Ψ.

4. VIX Option Pricing Using the COS Method

Traditional option pricing often uses inverse Fourier transforms, but the non-affine nature of the log-VIX creates theoretical barriers for the FFT method [23]. Therefore, this section employs the COS method [8] to price VIX options under stochastic interest rates and volatility. This approach uses Fourier-cosine expansions while maintaining exponential convergence speed and linear computational complexity.

Based on the SVJR model, we study a European call option on the VIX index with strike price Z, maturity T.

Its payoff at expiration is $\max(VIX_T - Z, 0)$ By Risk-neutral pricing, the option price C_t at time t is:

$$\begin{aligned} C_t &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\int_t^T R_s ds\right)\max(VIX_T - Z, 0)\right] \\ &= 100P(t,T)\mathbb{E}_t^{\mathbb{Q}}\left[\max\left(\sqrt{\delta_T^2} - Z', 0\right)\right] \\ &= 100P(t,T)\int_0^{+\infty}\max\left(\sqrt{x} - Z', 0\right) f_\tau(x|V_t,R_t,\lambda_t)dx \end{aligned} \quad (4.1)$$

where $P(t,t+\tau) = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\int_t^T R_s ds\right)\right]$, $\delta_T = \frac{VIX_T}{100}$, $\tau = T - t$, Z' is the strike price in percentage points, $f_\tau(x|V_t,R_t,\lambda_t)$ It is the risk-neutral conditional density function of δ_T given information at a time t. Using the inverse Fourier transform, this conditional density can be written as:

$$f_\tau(x|V_t,R_t,\lambda_t) = \frac{1}{2\pi}\int_{\mathbb{R}} e^{-iux}\varphi(u;V_t,R_t,\lambda_t,\tau)du \quad (4.2)$$

where $\varphi(u)$ is derived from Theorem 3.1 and Proposition 2.1 $\varphi(u;V_t,R_t,\lambda_t,\tau) = e^{iu\beta}\Psi(\varepsilon u, 0, \alpha u; V_t,R_t,\lambda_t,\tau)$.

This assumes $u_2 = 0, u_1 = \varepsilon u, u_3 = \alpha u$, here Ψ is the joint characteristic function from Theorem 3.1, ε,α,β Are the constant coefficients from Proposition 2.1?

After selecting a proper truncation range (explained later), $f_\tau(x|V_t,R_t,\lambda_t)$ it can be approximated using a truncated Fourier cosine series expansion.

$$\begin{aligned} &f_\tau(x|V_t,R_t,\lambda_t) \\ &\approx \frac{2}{b-a} \\ &\cdot \sum_{n=0}^{N-1}{}^{*} R\left[\varphi\left(\frac{n\pi}{b-a};V_t,R_t,\lambda_t,\tau\right)e^{-in\pi\frac{a}{b-a}}\right] \\ &\cdot \cos\left(n\pi\frac{x-a}{b-a}\right) \end{aligned} \quad (4.3)$$

where $\sum^*$ means the first term is multiplied by $\frac{1}{2}$, $\Re[\cdot]$ which is the operation taking the real part.

The pricing formula for VIX options is obtained by substituting (4.3) into (4.1) and interchanging integration and summation:

$$\begin{aligned} C_t &\approx 100P(t,T)\sum_{n=0}^{N-1}{}^{*}\Re \\ &\cdot\left[\varphi\left(\frac{n\pi}{b-a};V_t,R_t,\lambda_t,\tau\right)e^{-in\pi\frac{a}{b-a}}\right]A_n \end{aligned} \quad (4.4)$$

where

$$A_n = \frac{2}{b-a}\int_a^b \max\left(\sqrt{x} - Z', 0\right)\cos\left(n\pi\frac{x-a}{b-a}\right)dx$$

The following lemma shows that for VIX options, the payoff series coefficients A_n can be solved analytically:

Lemma 4.1: For the payoff function $\max\left(\sqrt{x} - Z', 0\right)$ on an interval $[a,b]$, the Fourier cosine series coefficients A_n Have these analytical solutions:

When $n = 0$,

$$A_n = \begin{cases} \frac{2}{b-a}\left(\frac{2}{3}b^{\frac{3}{2}} - Z'b + \frac{1}{3}Z'^3\right), & a < Z'^2, \\ \frac{2}{b-a}\left(\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right) - Z'(b-a)\right), & a \geq Z'^2. \end{cases} \quad (4.5)$$

When $n \neq 0$

$$A_n = \begin{cases} \frac{2}{b-a}\Re\left\{e^{-i\omega a}\left[\frac{\sqrt{b}-Z'}{i\omega}e^{i\omega b} + \frac{\sqrt{\pi}}{2\left(\sqrt{-i\omega}\right)^3}\left(erfz\left(\sqrt{b}\cdot\sqrt{-i\omega}\right) - erfz\left(Z'\cdot\sqrt{-i\omega}\right)\right)\right]\right\} & a < Z'^2, \\ \frac{2}{b-a}\Re\left\{e^{-i\omega a}\left[\frac{\sqrt{b}-Z'}{i\omega}e^{i\omega b} - \frac{\sqrt{a}-Z'}{i\omega} + \frac{\sqrt{\pi}}{2\left(\sqrt{-i\omega}\right)^3}\cdot\left(erfz\left(\sqrt{b}\cdot\sqrt{-i\omega}\right) - erfz\left(\sqrt{a}\cdot\sqrt{-i\omega}\right)\right)\right]\right\} & a \geq Z'^2. \end{cases} \quad (4.6)$$

where $\omega = \frac{n\pi}{b-a}$ the complex error function is defined as $erfz(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$.

Proof. To simplify notation, let $\omega = \frac{n\pi}{b-a}$. When $n \neq 0$

If $a < Z'^2$ the payoff function $\max\left(\sqrt{x} - Z', 0\right)$ has a non-zero interval $[Z'^2, b]$. At this time, the coefficient A_n

satisfies:

$$A_n = \frac{2}{b-a}\int_{Z'^2}^{b} \max(\sqrt{x} - Z', 0)\cos(\omega(x-a))dx \quad (4.7)$$

using the complex exponential form of cosine $\cos(x) = \Re[e^{ix}]$ ($\Re[\cdot]$ takes the real part). This converts the integral to complex exponential form:

$$A_n = \frac{2}{b-a}\Re\left[\int_{Z'^2}^{b} \sqrt{x}e^{i\omega(x-a)}dx - Z'\int_{Z'^2}^{b} e^{i\omega(x-a)}dx\right] \quad (4.8)$$

Using integration by parts and the complex error function, we solve the integral analytically:

$$\begin{aligned}&\int_{Z'^2}^{b} \sqrt{x}e^{i\omega(x-a)}dx \\ &= e^{-i\omega a}\int_{Z'^2}^{b} \sqrt{x}e^{i\omega x}dx \\ &= e^{-i\omega a}\left(\frac{\sqrt{b}}{i\omega}e^{i\omega b} - \frac{Z'}{i\omega}e^{i\omega Z'^2} - \int_{Z'}^{\sqrt{b}} \frac{1}{i\omega}e^{i\omega x^2}dx\right) \\ &\quad \cdot e^{-i\omega a}\left(erfz(\sqrt{b}\sqrt{-i\omega}) - erfz(Z'\sqrt{-i\omega})\right)\end{aligned} \quad (4.9)$$

Here $erfz(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$ is the complex error function. The second exponential integral directly gives:

$$\begin{aligned}Z'\int_{Z'^2}^{b} e^{i\omega(x-a)}dx &= Z'e^{-i\omega a}\int_{Z'^2}^{b} e^{i\omega x}dx \\ &= Z'e^{-i\omega a}\left(\frac{1}{i\omega}e^{i\omega b} - \frac{1}{i\omega}e^{i\omega Z'^2}\right)\end{aligned} \quad (4.10)$$

Substituting the integral results from (4.9) and (4.10) into the expression for A_n, we get the analytical solution for $n \neq 0$, $a \le Z'^2$

For $n = 0$ and $a \ge Z'^2$ similar integral derivations lead to the piecewise coefficient expression in (4.6).

When $n = 0$ and $a \ge Z'^2$ the payoff function $\max(\sqrt{x} - Z', 0)$ has a non-zero interval $[a, b]$

In this case, the coefficient A_n satisfies:

$$\begin{aligned}A_n &= \frac{2}{b-a}\int_a^b \max(\sqrt{x} - Z', 0) \\ &= \frac{2}{b-a}\int_a^b (\sqrt{x} - Z')\,dx \\ &= \frac{2}{b-a}\left(\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right) - Z'(b-a)\right)\end{aligned} \quad (4.11)$$

For the case $a < Z'^2$ $n = 0$, similar integral derivations yield the piecewise coefficient expression in (4.5).

In numerical integration, choosing a good truncation range balances speed and accuracy. Following [8]'s framework but adjusting for δ_T^2 (since $\delta_T = \frac{VIX_T}{100}$ it must be non-negative), we set the lower bound to avoid negative values. The truncation range $[a, b]$ is:

$$[a, b] \triangleq \left[\max\left(c_1 - L\sqrt{c_2 + \sqrt{c_4}}, 0\right), c_1 + L\sqrt{c_2 + \sqrt{c_4}}\right]$$

Here $L = 10$ is an empirically set truncation parameter.

c_1, c_2, c_3 Are the first, second, and fourth-order cumulants of δ_T^2 . For δ_T^2 its-th order cumulant c_k is defined by differentiating the log-characteristic function:

$$c_k = \frac{1}{i^k}\frac{\partial^k(\log\varphi(u))}{\partial u^k}\bigg|_{u=0} \quad (4.12)$$

Where $\varphi(u)$ is the characteristic function of δ_T^2 and $i = \sqrt{-1}$ is the imaginary unit.

Lemma 4.2: Suppose the random variable δ_T^2 follows the stochastic process described in Proposition 3.1. Then its first four cumulants have these analytical forms:

$$\begin{aligned}c_1 &= \beta + \varepsilon e^{-\kappa_v\tau}V_t + \alpha e^{-(\kappa_\lambda-\eta_\lambda)\tau}\lambda_t + \varepsilon\theta_v\left(\tau - \frac{1-e^{-\kappa_v\tau}}{\kappa_v}\right) \\ &\quad + \alpha\theta_\lambda\kappa_\lambda\left(\frac{1}{(\kappa_\lambda-\eta_\lambda)^2}\left((\kappa_\lambda-\eta_\lambda)\tau + e^{-(\kappa_\lambda-\eta_\lambda)\tau} - 1\right)\right)\end{aligned}$$

$$\begin{aligned}c_2 &= \frac{\eta_v^2\varepsilon^2}{\kappa_v^3}\left(e^{-\kappa_v\tau} - 4e^{-2\kappa_v\tau} + 3e^{-3\kappa_v\tau}\right)V_t \\ &\quad + \frac{\eta_\lambda^2\alpha^2}{(\kappa_\lambda-\eta_\lambda)^3}\left(e^{-(\kappa_\lambda-\eta_\lambda)\tau} - 4e^{-2(\kappa_\lambda-\eta_\lambda)\tau} + 3e^{-3(\kappa_\lambda-\eta_\lambda)\tau}\right)\lambda_t \\ &\quad + \frac{\eta_v^2\varepsilon^2\theta_v}{2\kappa_v^3}\left(1 - 4e^{-\kappa_v\tau} + 6e^{-2\kappa_v\tau} - 3e^{-3\kappa_v\tau}\right) \\ &\quad + \frac{\eta_\lambda^2\alpha^2\theta_\lambda\theta\kappa_\lambda}{(\kappa_\lambda-\eta_\lambda)^4}\left(1 - 4e^{-(\kappa_\lambda-\eta_\lambda)\tau} + 6e^{-2(\kappa_\lambda-\eta_\lambda)\tau} - 3e^{-3(\kappa_\lambda-\eta_\lambda)\tau}\right)\end{aligned}$$

$$\begin{aligned}c_4 &= \frac{3\varepsilon^4\eta_v^4}{\kappa_v^5}\left(e^{-\kappa_v\tau} - 11e^{-2\kappa_v\tau} + 24e^{-3\kappa_v\tau} - 15e^{-4\kappa_v\tau}\right)V \\ &\quad + \frac{3\alpha^4\eta_\lambda^4}{(\kappa_\lambda-\eta_\lambda)^5}\left(e^{-(\kappa_\lambda-\eta_\lambda)\tau} - 11e^{-2(\kappa_\lambda-\eta_\lambda)\tau} + 24e^{-3(\kappa_\lambda-\eta_\lambda)\tau}\right. \\ &\quad \left. -15e^{-4(\kappa_\lambda-\eta_\lambda)\tau}\right)\lambda_t + \frac{\varepsilon^4\eta_v^4\theta_v}{4\kappa_v^5}\left(3 - 33e^{-\kappa_v\tau} + 108e^{-2\kappa_v\tau}\right. \\ &\quad \left. -135e^{-3\kappa_v\tau} + 57e^{-4\kappa_v\tau}\right) + \frac{\alpha^4\eta_\lambda^4\theta_z\kappa_z}{(\kappa_\lambda-\eta_\lambda)^6}\left(3 - 33e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right. \\ &\quad \left. +108e^{-2(\kappa_\lambda-\eta_\lambda)\tau} - 135e^{-3(\kappa_\lambda-\eta_\lambda)\tau} + 57e^{-4(\kappa_\lambda-\eta_\lambda)\tau}\right) \\ &\quad + \alpha^2\eta_v^2\eta_\lambda^2\left(\frac{12\theta_v}{\kappa_v^4}\left(1 - e^{-\kappa_v\tau}\right) + \frac{12\kappa_\lambda\theta_\lambda}{(\kappa_\lambda-\eta_\lambda)^4}\right. \\ &\quad \cdot\left(1 - e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right) + \frac{6V_t}{\kappa_v^3}\left(e^{-\kappa_v\tau} - 4e^{-2\kappa_v\tau} + 3e^{-3\kappa_v\tau}\right) \\ &\quad + \frac{6\lambda_t}{(\kappa_\lambda-\eta_\lambda)^3}\left(e^{-(\kappa_\lambda-\eta_\lambda)\tau} - 4e^{-2(\kappa_\lambda-\eta_\lambda)\tau} + 3e^{-3(\kappa_\lambda-\eta_\lambda)\tau}\right) \\ &\quad + \frac{24\theta_v}{\kappa_v^3}\left(e^{-\kappa_v\tau} - e^{-2\kappa_v\tau}\right) + \frac{24\kappa_\lambda\theta_\lambda}{(\kappa_\lambda-\eta_\lambda)^3}\left(e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right. \\ &\quad \left. -e^{-2(\kappa_\lambda-\eta_\lambda)\tau}\right) + \frac{18\lambda_t}{\kappa_v^2(\kappa_\lambda-\eta_\lambda)}\left(e^{-\kappa_v\tau}e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right) \\ &\quad + \frac{18\kappa_\lambda\theta_\lambda}{\kappa_v(\kappa_\lambda-\eta_\lambda)^2}\left(e^{-\kappa_v\tau}e^{-(\kappa_\lambda-\eta_\lambda)\tau}\right) + \frac{12\theta_v}{\kappa_v(\kappa_\lambda-\eta_\lambda)} \\ &\quad \left.\cdot\left(1 - e^{-(\kappa_v+\kappa_\lambda-\eta_\lambda)\tau}\right) + \frac{12\kappa_\lambda\theta_\lambda}{\kappa_v(\kappa_\lambda-\eta_\lambda)^2}\left(1 - e^{-(\kappa_v+\kappa_\lambda-\eta_\lambda)\tau}\right)\right)\end{aligned}$$

The proof is omitted here. By combining the conclusions of the above two lemmas, we can directly present the following proposition, and its proof is also omitted here.

Proposition 4.3: Under the SVJR model defined in Equation (1), given a truncation range $[a, b]$, the price at time t of a VIX call option with strike price Z and maturity T can be approximated by:

$$C_t \approx 100P(t,T)\sum_{n=0}^{N-1}{}^{*}\Re\left[\varphi\left(\frac{n\pi}{b-a}; V_t, R_t, \lambda_t, \tau\right)e^{-in\pi\frac{a}{b-a}}\right]A_n$$

Here $\tau = T - t$ is the time to maturity of the option. $\Psi(\cdot)$ Is the conditional characteristic function defined in Proposition 3.1? The coefficients A_n come from Lemma 4.1, $\Re[\cdot]$ which takes the real part of complex numbers.

For a VIX futures contract maturing at T, its price F_t

At time t follows risk-neutral pricing:

$$F_t = \mathbb{E}_t^{\mathbb{Q}}(VIX_T) = 100\mathbb{E}_t^{\mathbb{Q}}\left(\sqrt{\delta_T^2}\right)$$

This means the VIX futures price is the risk-neutral expectation of the VIX index. It comes from taking the square root of δ_T^2 and scaling by 100.

5. Conclusion

This paper proposes a three-factor VIX option pricing model incorporating stochastic interest rates, stochastic volatility, and stochastic jump intensity. Theoretically, it pioneers the quantification of monetary policy's transmission effect on market panic through an interest rate-jump coupling parameter. Methodologically, it enhances the COS algorithm to improve computational efficiency. Empirical results demonstrate that the model reduces pricing errors while exhibiting stronger robustness during policy adjustments and reveals significant interaction effects between interest rate parameters and jump decay rates. Future research may extend this model to more complex derivatives such as VIX futures options, further enriching the theory and practice of volatility derivatives pricing.

References

[1] Bekaert, Geert, Marie Hoerova, and Marco Lo Duca. "Risk, uncertainty and monetary policy." Journal of Monetary Economics 60.7: 771-788,2013.

[2] Sattayatham, P., and S. Pinkham. "Option pricing for a stochastic volatility Lévy model with stochastic interest rates." Journal of the Korean Statistical Society 42.1: 25-36,2013.

[3] Font, Oriol Zamora. "Pricing VIX options under the Heston-Hawkes stochastic volatility model." arxiv preprint arxiv:2406.13508,2024.

[4] Jing, Bo, Shenghong Li, and Yong Ma. "Consistent pricing of VIX options with the Hawkes jump-diffusion model." The North American Journal of Economics and Finance 56: 101326,2021.

[5] Dima, Bogdan, and Ştefana Maria Dima. "The non-linear impact of monetary policy on shifts in economic policy uncertainty: evidence from the United States of America." Empirica 51.3: 755-781,2024.

[6] Eraker, Bjørn. "Do stock prices and volatility jump? Reconciling evidence from spot and option prices." The Journal of Finance 59.3: 1367-1403,2004.

[7] Broadie, Mark, Mikhail Chernov, and Michael Johannes. "Model specification and risk premia: Evidence from futures options." The Journal of Finance 62.3: 1453-1490,2007.

[8] Fang, Fang, and Cornelis W. Oosterlee. "A novel pricing method for European options based on Fourier-cosine series expansions." SIAM Journal on Scientific Computing 31.2: 826-848, 2009.

[9] Zhang, Baocheng, and Cornelis W. Oosterlee. "Pricing of early-exercise Asian options under Lévy processes based on Fourier cosine expansions." Applied Numerical Mathematics 78: 14-30,2014.

[10] Wang, Zhiguang, and Robert T. Daigler. "The performance of VIX option pricing models: empirical evidence beyond simulation." Journal of Futures Markets 31.3: 251-281,2011.

[11] Merton, Robert C. "Option pricing when underlying stock returns are discontinuous." Journal of Financial Economics 3.1-2: 125-144,1976.

[12] Hawkes, Alan G. "Spectra of some self-exciting and mutually exciting point processes." Biometrika 58.1: 83-90,1971.

[13] Bacry, Emmanuel, and Jean-François Muzy. "Hawkes model for price and trades high-frequency dynamics." Quantitative Finance 14.7: 1147-1166,2014.

[14] Aït-Sahalia, Yacine, Julio Cacho-Diaz, and Roger JA Laeven. "Modeling financial contagion using mutually exciting jump processes." Journal of Financial Economics 117.3: 585-606,2015.

[15] Zhang, Ruizhe, Cuixiang Li, and Huili Liu. "The pricing of forward start options under the jump diffusion model with stochastic interest rate and stochastic volatility." Communications in Statistics-Theory and Methods: 1-20,2025.

[16] Davison, Anthony C., and Richard L. Smith. "Models for exceedances over high thresholds." Journal of the Royal Statistical Society Series B: Statistical Methodology 52.3: 393-425,1990.

[17] Ahn, Dong-Hyun, and Bin Gao. "A parametric nonlinear model of term structure dynamics." The Review of Financial Studies 12.4: 721-762,1999.

[18] Bakshi, Gurdip, and Zhiwu Chen. "Stock valuation in dynamic economies." Journal of Financial Markets 8.2: 111-151,2005.

[19] Duffie, Darrell, Jun Pan, and Kenneth Singleton. "Transform analysis and asset pricing for affine jump‐diffusions." Econometrica 68.6: 1343-1376,2000.

[20] Kaeck, Andreas, Vincent van Kervel, and Norman J. Seeger. "Price impact versus bid–ask spreads in the index option market." Journal of Financial Markets 59: 100675,2022.

[21] Christoffersen, Peter, and Kris Jacobs. "Which GARCH model for option valuation?" Management science 50.9 (2004): 1204-1221,2004.

[22] Erel, Isil, and Jack Liebersohn. "Can FinTech reduce disparities in access to finance? Evidence from the Paycheck Protection Program." Journal of Financial Economics 146.1: 90-118,2022.

[23] Carr, Peter, and Dilip Madan. "Option valuation using the fast Fourier transform." Journal of Computational Finance 2.4: 61-73, 1999.

[24] Tour, G., Thakoor, N., Khaliq, A., & Tangman, D. "COS method for option pricing under a regime-switching model with time-changed Lévy processes". Quantitative Finance, 18(4), 673-692, 2018.

[25] Bardgett, Chris, Elise Gourier, and Markus Leippold. "Inferring volatility dynamics and risk premia from the S&P 500 and VIX markets." Journal of Financial Economics 131.3: 593-618,2019.

[26] Zhu, Song, and Guang‐Hua Lian. "An analytical formula for VIX futures and its applications." Journal of Futures Markets 32.2: 166-190,2012.

[27] Lian, Guang-Hua, and Song Zhu. "Pricing VIX options with stochastic volatility and random jumps." Decisions in Economics and Finance 36 (2013): 71-88,2013.

[28] Chau, Kwok-Wing, Sheung Chi Phillip Yam, and Hailiang Yang. "Fourier-cosine method for ruin probabilities." Journal of Computational and Applied Mathematics 281: 94-106,2015.
[29] Jaimungal, Sebastian, and Tao Wang. "Catastrophe options with stochastic interest rates and compound Poisson losses." Insurance: Mathematics and Economics 38.3: 469-483,2006.
[30] Zhu, Yingzi, and E. Zhang. "Variance term structure and VIX futures pricing." International Journal of Theoretical and Applied Finance 10.01: 111-127,2007.
[31] Duffie, Darrell, Jun Pan, and Kenneth Singleton. "Transform analysis and asset pricing for affine jump‐diffusions." Econometrica 68.6: 1343-1376,2000.

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