DOI: 10.53469/jgebf.2025.07(06).03

Exchange Option Pricing under the Mixed-Exponential Jump Diffusion Model

Shuangfei Wei

College of Mathematics and Statistics, Guangxi Normal University, Guangxi 541004, Guilin, China

Abstract: ***This paper considers a pricing problem on a kind of Exotic option: Exchange options under a mixed-exponential jump diffusion model within the stochastic interest rate and stochastic volatility framework. By applying the Feynman-Kac theorem, the joint characteristic function, and Fourier inverse transformation techniques, the semi-analytical pricing formula for the option is obtained. This research provides critical theoretical foundations and empirical insights for pricing related financial derivatives and managing associated risks.***

Keywords: Stochastic interest rate, Stochastic volatility, Mixed-exponential jump diffusion model, Exchange option.

1. Introduction

As an exotic option, the exchange option [1] grants the holder the right to exchange two underlying assets at the expiration date. Its pricing mechanism is significantly more complex than that of single-asset options due to the correlation between the two assets. The classic Black-Scholes model [2] (BS model) provides a theoretical foundation for option pricing, but its core assumptions—that asset prices follow geometric Brownian motion, implied volatility and a constant risk-free interest rate—are inconsistent with empirical observations such as the “sharp peaks and fat tails” distribution of asset returns, the “volatility smile or skew” phenomenon, and the time-varying nature of interest rates. To reduce the gap between theory and market reality, academia has systematically improved pricing models from three dimensions: 1) Jump diffusion models, including the Merton jump diffusion model [3], double-index jump diffusion model [4], and mixed-exponential jump diffusion model [5] (MEM); 2) Stochastic volatility models, such as the Hull-White stochastic volatility model [6], Heston stochastic volatility model [7], and double Heston stochastic volatility model [8]; 3) Stochastic interest rate models, including the Vasicek model [9] and CIR stochastic interest rate model [10]. However, single models have limitations in capturing multidimensional risk factors. Therefore, scholars have combined single stochastic volatility models and stochastic interest rate models with jump diffusion models to improve pricing accuracy. Li et al. [11] provided a risk-neutral pricing for exchange options under the jump diffusion model; Kim and Park [12] derived the Margrabe formula for exchange options under the Heston model; Cheang and Graces [13] combined the stochastic volatility model with the jump diffusion model to study the pricing of European and American exchange options.

Research by scholars like Bates [14], Bakshi et al. [15] shows that combining stochastic volatility, stochastic interest rate, and jump diffusion models works much better than using a single model for option pricing and fits market features more accurately. The Merton model [3] captures the "sharp peaks and fat tails" in asset returns well but misses the asymmetry in asset prices. The Lévy model fits these features too [16][17], but as Cai and Kou [5] noted, it struggles to show both long- and short-term patterns at the same time, so they came up with a mixed-exponential jump diffusion model. Current research on exchange option pricing has not looked at combining the double Heston stochastic volatility model with the mixed-exponential jump diffusion model. Since the Vasicek model can have negative interest rates, this paper introduces a new mixed-exponential jump diffusion model with stochastic interest rates and stochastic volatility, where volatility follows a CIR model and interest rates are shown as a double exponential. Using tools like joint characteristic functions, the Feynman-Kac theorem, and Fourier inverse transform, we derive the pricing formula for exchange options.

2. Model Specification

Given a complete probability space $(\Omega,\mathcal{F},Q)$, where $(\mathcal{F}_t)_{0\le t\le T}$ represents the information flow under normal conditions. Assume that the probability measure Q is the equivalent martingale measure. Suppose that there are only two types of risky assets in the market: S_{1t}, S_{2t}, whose corresponding values $X_{1t}=\ln S_{1t}$, $X_{2t}=\ln S_{2t}$ and long-term and short-term volatilities V_{1t} and V_{2t} satisfy the following system of stochastic differential equations under the measure Q:

$$\begin{cases} dX_{1t}=\left(R_t-\frac{1}{2}\sigma_{11}^2V_{1t}-\frac{1}{2}\sigma_{12}^2V_{2t}-\lambda\kappa_1\right)dt+\sigma_{11}\sqrt{V_{1t}}dW_{1t}^{s_1} \\ \qquad +\sigma_{12}\sqrt{V_{2t}}dW_{2t}^{s_1}+J_{1t}^{s}dN_{1t}^{s}+J_{1t}^{c}dN_t^{c}, \\ dX_{2t}=\left(R_t-\frac{1}{2}\sigma_{21}^2V_{1t}-\frac{1}{2}\sigma_{22}^2V_{2t}-\lambda\kappa_2\right)dt+\sigma_{21}\sqrt{V_{1t}}dW_{1t}^{s_2} \\ \qquad +\sigma_{22}\sqrt{V_{2t}}dW_{2t}^{s_2}+J_{2t}^{s}dN_{2t}^{s}+J_{2t}^{c}dN_t^{c}, \\ dV_{1t}=\beta_1(\theta_{v1}-V_{1t})dt+\sigma_1\sqrt{V_{1t}}dW_{1t}^{v}, \\ dV_{2t}=\beta_2(\theta_{v2}-V_{2t})dt+\sigma_2\sqrt{V_{2t}}dW_{2t}^{v}. \end{cases} \tag{1}$$

$W_t=\{W_{1t}^{s_1},W_{2t}^{s_1},W_{1t}^{s_2},W_{2t}^{s_2},W_{1t}^{v},W_{2t}^{v}\}$ represents a 6-dimensional standard Brownian motion, and $N_t=\{N_{1t}^{s},N_{2t}^{s},N_t^{c}\}$ is a 3-dimensional Poisson process with intensity parameters $(\lambda_1,\lambda_2,\lambda_c)$. Assume the correlation coefficients $Corr(W_{lt}^{s},W_{jt}^{v})=\rho_{lj}$, $Corr(W_{lt}^{s_1},W_{lt}^{s_2})=\rho_l(l,j=1,2)$. where ρ_{lj}, ρ_l are constants, and the remaining Brownian motions are independent of each other. β_i, θ_{vi}, $\sigma_i(i=1,2)$ are non-negative constants, and satisfy $2\beta_i\theta_{vi}\ge\sigma_i^2$. β_i, θ_{vi}, σ_i are called the mean reversion rate, long-term equilibrium level, and standard deviation of V_{it}, respectively. $R_t=\vartheta_0+\vartheta_1V_{1t}+\vartheta_2V_{2t}$, where ϑ_0, ϑ_1, ϑ_2 are constants.

$Y_i = J_{it}^s$ is a random variable following a mixed exponential distribution, with probability density function:

$$f_{Y_1}(y) = p_{u_1} \sum_{k_1=1}^{m} p_{k_1} \eta_{k_1} e^{-\eta_{k_1} y} I_{\{y \geq 0\}} + q_{d_1} \sum_{k_2=1}^{n} q_{k_2} \theta_{k_2} e^{\theta_{k_2} y} I_{\{y<0\}},$$
$$f_{Y_2}(y) = p_{u_2} \sum_{k_3=1}^{m} p_{k_3} \eta_{k_3} e^{-\eta_{k_3} y} I_{\{y \geq 0\}} + q_{d_2} \sum_{k_4=1}^{n} q_{k_4} \theta_{k_4} e^{\theta_{k_4} y} I_{\{y<0\}}, \quad (2)$$

the up-jump probability satisfies $1 \geq p_{u_i} \geq 0$, the down-jump probability $q_{d_i} = 1 - p_{u_i}$, and the probabilities of up and down jumps $p_{k_1}, q_{k_2}, p_{k_3}, q_{k_4} \in (-\infty, \infty)$, with $\sum_{k_1=1}^{m} p_{k_1} = \sum_{k_2=1}^{n} q_{k_2} = \sum_{k_3=1}^{m} p_{k_3} = \sum_{k_4=1}^{n} q_{k_4} = 1$. $\eta_{k_1}, \theta_{k_2}, \eta_{k_3}, \theta_{k_2}$ represent the jump magnitudes for up and down jumps, respectively, and satisfy the following conditions:

(1) $\eta_{k_1} \geq 1, \theta_{k_2} \geq 0, \eta_{k_3} \geq 1, \theta_{k_4} \geq 0, \forall k_1, k_3 = 1, \dots, m$, $\forall k_2, k_4 = 1, \dots, n$.
(2) $\sum_{k_1=1}^{m_1} p_{k_1} \eta_{k_1} \geq 0, \sum_{k_2=1}^{n_1} q_{k_2} \theta_{k_2} \geq 0, \forall m_1 = 1, \dots, m$, $\forall n_1 = 1, \dots, n$.
(3) $\sum_{k_3=1}^{m_2} p_{k_3} \eta_{k_3} \geq 0, \sum_{k_4=1}^{n_2} q_{k_4} \theta_{k_4} \geq 0, \forall m_2 = 1, \dots, m$, $\forall n_2 = 1, \dots, n$.

Assume that W_t, J_t, N_t are independent of each other, and the joint jump amplitude distribution of the jump process is represented by the variable v. Assume that v has a jump transformation:

$$v(c_1, c_2) = \frac{\lambda_1 v_1(c_1) + \lambda_2 v_2(c_2)}{\lambda}. \quad (3)$$

Here,

$$\lambda = \lambda_1 + \lambda_2, \kappa_1 = v(1,0) - 1, \kappa_2 = v(0,1) - 1,$$
$$v_1(c) = p_{u_1} \sum_{k_1}^{m} \frac{p_{k_1} \eta_{k_1}}{\eta_{k_1} - c} + q_{d_1} \sum_{k_2}^{n} \frac{q_{k_2} \theta_{k_2}}{\theta_{k_2} + c},$$
$$v_2(c) = p_{u_2} \sum_{k_3}^{m} \frac{p_{k_3} \eta_{k_3}}{\eta_{k_3} - c} + q_{d_2} \sum_{k_4}^{n} \frac{q_{k_4} \theta_{k_4}}{\theta_{k_4} + c}.$$

The simplified models (1)-(3) represent the MEJ-2Heston-CIR model (a double Heston-CIR stochastic mixed- exponential jump diffusion model)

3. Main Results

3.1 Characteristic Function

Let $\psi(t, \boldsymbol{X}_T, \boldsymbol{V}_T, \boldsymbol{u}_x, \boldsymbol{u}_v, T)$ be the joint conditional characteristic function of the random vectors $(X_{1t}, X_{2t}, V_{1t}, V_{2t})$ under the measure Q based on the reference family $\mathcal{F}_t$.

$$\psi(t, \boldsymbol{X}_T, \boldsymbol{V}_T, \boldsymbol{u}_x, \boldsymbol{u}_v, T) = E_t^Q[e^{-\int_t^T R_s ds + i\boldsymbol{u}_x \boldsymbol{X}_T^{\perp} + i\boldsymbol{u}_v \boldsymbol{V}_T^{\perp}}], \quad (4)$$

The vectors $\boldsymbol{X}_T = (X_{1t}, X_{2t})$, $\boldsymbol{V}_T = (V_{1t}, V_{2t})$, $\boldsymbol{u}_x = (u_{11}, u_{12})$, $\boldsymbol{u}_v = (u_{21}, u_{22})$, $\boldsymbol{x} = (x_1, x_2)$, $\boldsymbol{v} = (v_1, v_2)$. $\boldsymbol{X}_T^{\perp}$, $\boldsymbol{V}_T^{\perp}$ represent the rotations of $\boldsymbol{X}_T$, $\boldsymbol{V}_T$ respectively, with $u_{lj} \in \mathbb{C}(l, j = 1,2)$. Denote (4) as $\psi(t, u_{11}, u_{12}, u_{21}, u_{22}, T)$.

Theorem 1. Suppose S_{1t} and S_{2t} satisfy the MEJ-2Heston-CIR model, then the characteristic function $\psi(t, u_{11}, u_{12}, u_{21}, u_{22}, T)$ has the following analytical expression:

$$\psi(t, \boldsymbol{X}_T, \boldsymbol{V}_T, \boldsymbol{u}_x, \boldsymbol{u}_v, T) = exp(iu_{11}x_1 + iu_{12}x_2 + \sum_{i=1}^{2} A_i(\tau, \boldsymbol{u}_x, \boldsymbol{u}_v) v_i + B(\tau, \boldsymbol{u}_x, \boldsymbol{u}_v)),$$

Here $\tau = T - t$,

$$\begin{cases} A_i(\tau, \boldsymbol{u}_x, \boldsymbol{u}_v) = \frac{1}{\sigma_i^2}\left[a_i(\boldsymbol{u}_x) + \gamma_i(\boldsymbol{u}_x) - \frac{2\gamma_i(\boldsymbol{u}_x)}{1 - g_i(\boldsymbol{u}_x, \boldsymbol{u}_v) \times e^{-\gamma_i(\boldsymbol{u}_x)\tau}}\right], \\ B(\tau, \boldsymbol{u}_x, \boldsymbol{u}_v) = [\sum_{l=1}^{2}[\vartheta_0 - \lambda\kappa_l] iu_{1l} + \sum_{l=1}^{2} \lambda_l v_l(iu_{1l}) + \lambda v(iu_{11}, iu_{12}) - \lambda - \vartheta_0]\tau \\ \quad + \sum_{i=1}^{2} \frac{\beta_i \theta_{vi}}{\sigma_i^2}\left[(a_i(\boldsymbol{u}_x) - \gamma_i(\boldsymbol{u}_x))\tau - 2\ln \frac{1 - g_i(\boldsymbol{u}_x, \boldsymbol{u}_v) \times e^{-\gamma_i(\boldsymbol{u}_x)\tau}}{1 - g_i(\boldsymbol{u}_x, \boldsymbol{u}_v)}\right], \\ a_i(\boldsymbol{u}_x) = \beta_i - i\rho_{11}\sigma_{1i}\sigma_i u_{11} - i\rho_{21}\sigma_{2i}\sigma_i u_{12}, \\ b_i(\boldsymbol{u}_x) = \sum_{l=1}^{2}\left[\left(\vartheta_i - \frac{1}{2}\sigma_{li}\right) iu_{1l} - \frac{1}{2}\sigma_{li}^2 {u_{1l}}^2\right] - \vartheta_i - \rho_i \sigma_{1j}\sigma_{2j} u_{11} u_{12}, \\ \gamma_i(\boldsymbol{u}_x) = \sqrt{a_i(\boldsymbol{u}_x)^2 - 2\sigma_i^2 b_i(\boldsymbol{u}_x)}, \\ g_i(\boldsymbol{u}_x, \boldsymbol{u}_v) = \frac{i\sigma_i^2 u_{2i} - a_i(\boldsymbol{u}_x) + \gamma_i(\boldsymbol{u}_x)}{i\sigma_i^2 u_{2i} - a_i(\boldsymbol{u}_x) - \gamma_i(\boldsymbol{u}_x)}. \end{cases}$$

Proof: From the $It\hat{o}$ formula, we can obtain:

$$\begin{cases} \frac{\partial \psi}{\partial t} + \sum_{l=1}^{2}(\vartheta_0 + \vartheta_1 v_1 + \vartheta_2 v_2 - \frac{1}{2}\sigma_{l1}^2 - \frac{1}{2}\sigma_{l2}^2 - \lambda\kappa_l) \frac{\partial \psi}{\partial x_l} \\ + \sum_{l=1}^{2}(\frac{1}{2}\sigma_{l1}^2 + \frac{1}{2}\sigma_{l2}^2) \frac{\partial^2 \psi}{\partial x_l^2} + \sum_{i=2}^{2}\left[\beta_i(\theta_{vi} - v_i)\frac{\partial \psi}{\partial v_i} + \frac{1}{2}\sigma_i^2 v_i \frac{\partial^2 \psi}{\partial v_i^2}\right] \\ + \sum_{l=1}^{2} \rho_l \sigma_{1l}\sigma_{2l} v_l \frac{\partial^2 \psi}{\partial x_1 \partial x_2} + \sum_{l=1}^{2}\sum_{i=1}^{2} \rho_{li}\sigma_{li}\sigma_i v_i \frac{\partial^2 \psi}{\partial x_l \partial v_i} - (\vartheta_0 + \vartheta_1 v_1 + \vartheta_2 v_2)\psi \\ + \lambda_1 E[\psi(t, x_1 + J_{1t}^s, x_2, \boldsymbol{v}, \boldsymbol{u}_x, \boldsymbol{u}_v, T) - \psi(t, \boldsymbol{x}, \boldsymbol{v}, \boldsymbol{u}_x, \boldsymbol{u}_v, T)] \\ + \lambda_2 E[\psi(t, x_1, x_2 + J_{2t}^s, \boldsymbol{v}, \boldsymbol{u}_x, \boldsymbol{u}_v, T) - \psi(t, \boldsymbol{x}, \boldsymbol{v}, \boldsymbol{u}_x, \boldsymbol{u}_v, , T)] \\ + \lambda_c E[\psi(t, x_1 + J_{1t}^c, x_2 + J_{2t}^c, \boldsymbol{v}, \boldsymbol{u}_x, \boldsymbol{u}_v, T) - \psi(t, \boldsymbol{x}, \boldsymbol{v}, \boldsymbol{u}_x, \boldsymbol{u}_v, T)] = 0, \\ \psi(T, \boldsymbol{X}_T, \boldsymbol{V}_T, \boldsymbol{u}_x, \boldsymbol{u}_v, T) = e^{iu_{11}X_{1T} + iu_{12}X_{2T} + iu_{21}V_{1T} + iu_{22}V_{2T}}. \end{cases} \quad (5)$$

From the references [18], it is known that $\psi(t, u_{11}, u_{12}, u_{21}, u_{22}, T)$ has a reflective structure, Then, according to the above equation, we can get

$$\begin{cases} \frac{\partial A_i(\tau)}{\partial \tau} = \frac{1}{2}\sigma_i^2 A_i^2(\tau) - a_i(\boldsymbol{u}_x) A_i(\tau) + b_i(\boldsymbol{u}_x) = 0, \\ A_i(0, \boldsymbol{u}_x, \boldsymbol{u}_v) = iu_{2i}. \end{cases} \quad (6)$$

$$\begin{cases} \frac{\partial B(\tau)}{\partial \tau} = \sum_{l=1}^{2}[\vartheta_0 - \lambda\kappa_l] iu_{1l} + \sum_{i=1}^{2} \beta_i \theta_{vi} A_i(\tau) - \vartheta_0 + \sum_{l=1}^{2} \lambda_l v_l(iu_{1l}) + \lambda v(iu_{11}, iu_{12}) - \lambda, \\ B(0, \boldsymbol{u}_x, \boldsymbol{u}_v) = 0. \end{cases} \quad (7)$$

$A_i(t,\boldsymbol{u}_x,\boldsymbol{u}_v,T)=A_i(\tau,\boldsymbol{u}_x,\boldsymbol{u}_v)=A_i(\tau)$, $B(t,\boldsymbol{u}_x,\boldsymbol{u}_v,T)=B(\tau,\boldsymbol{u}_x,\boldsymbol{u}_v)=B(\tau)$. By integrating both sides of equation (6), we get

$$A_i(\tau,\boldsymbol{u}_x,\boldsymbol{u}_v)=\frac{1}{\sigma_i^2}\left[a_i(\boldsymbol{u}_x)+\gamma_i(\boldsymbol{u}_x)-\frac{2\gamma_i(\boldsymbol{u}_x)}{1-g_i(\boldsymbol{u}_x,\boldsymbol{u}_v)\times e^{-\gamma_i(\boldsymbol{u}_x)\tau}}\right], \quad (8)$$

Integrating both sides of (8) over the interval $[0,\tau]$ yields, we get

$$\int_0^\tau A_i(s)ds=\frac{1}{\sigma_i^2}\left[\left(a_i(\boldsymbol{u}_x)-\gamma_i(\boldsymbol{u}_x)\right)\tau-ln\frac{1-g_i(\boldsymbol{u}_x,\boldsymbol{u}_v)\times e^{-\gamma_i(\boldsymbol{u}_x)\tau}}{1-g_i(\boldsymbol{u}_x,\boldsymbol{u}_v)}\right], \quad (9)$$

By substituting (9) into (7), we can obtain $B(\tau)$.

3.2 Exchange Option Pricing

According to the definition of exchange options, the value of this option at maturity date T is

$$C_D(T,S_{1t},S_{2t},T)=(S_{1T}-S_{2T})^+. \quad (10)$$

Thus, the value of the exchange option under the risk-neutral measure Q at time $t\in[0,T]$ is

$$\begin{aligned}C_D(t,S_{1t},S_{2t},T)&=E\left[e^{-\int_t^T R_s ds}S_{1T}I_{(S_{1T}>S_{2T})}\right]-E\left[e^{-\int_t^T R_s ds}S_{2T}I_{(S_{1T}>S_{2T})}\right]\\&=S_{1t}Q_1(X_{1T}>X_{2T})-S_{2t}Q_2(X_{1T}>X_{2T}),\end{aligned} \quad (11)$$

Q_1, Q_2 are two probability measures,

$$\frac{dQ_i}{dQ}|F_t=e^{-\int_t^T R_s ds}\frac{S_{iT}}{S_{it}},$$

It can be easily calculated that

$$E\left[e^{-\int_t^T R_s ds}\frac{S_{iT}}{S_{it}}\right]=1,$$

It can be calculated using the inverse Fourier transform formula as follows:

$$\begin{aligned}Q_i(ln\,S_{1T}\geq ln\,S_{2T})&=\Pi_i(ln\,S_{1t},ln\,S_{2t})\\&=\frac{1}{2}+\frac{1}{\pi}\int_0^\infty\Re\left[\frac{\phi_i(u)}{iu}\right]du,\end{aligned}$$

where $\Re(z)$ represents the real part of $z\in\mathbb{C}$. Since there is a unique relationship between the distribution function and the characteristic function, the corresponding characteristic function can be obtained.

$$\phi_1(u)=\frac{\psi(t,u-i,-u,0,0,T)}{\psi(t,-i,0,0,0,T)},\ \phi_2(u)=\frac{\psi(t,u,-u-i,0,0,T)}{\psi(t,0,-i,0,0,T)}.$$

Theorem 2. Suppose S_{1t} and S_{2t} follow the MEJ-2Heston-CIR model. The price of the exchange option with maturity T at time $t\in[0,T]$ is

$$C_D(t,S_{1t},S_{2t},T)=S_{1t}\Pi_1(ln\,S_{1t},ln\,S_{2t})-S_{2t}\Pi_2(ln\,S_{1t},ln\,S_{2t}). \quad (12)$$

4. Conclusion

This paper studies the pricing problem of exchange options under the 2Heston-CIR mixed exponential jump diffusion model. By using the $It\hat{o}$ formula, Feynman-Kac theorem, and Fourier transform methods, the pricing formula for exchange option is derived. The research results in this paper can be extended to the pricing of American options or other exotic options.

References

[1] Margrabe W, "The value of an option to exchange one asset for another", Journal of Finance, vol. 33, no. 1, pp. 177-186, 1978.

[2] Black F, Scholes M, "The pricing of options and corporate liabilities", Journal of Political Economy, vol. 83, no. 3, pp. 637-654, 1973.

[3] Merton R C, "Option pricing when underlying stock returns are discontinuous", Journal of Financial Economics, vol. 3, pp. 125-144, 1976.

[4] Kou S, "A jump-diffusion model for option pricing", Management Science, vol. 48, no. 8, pp. 1086-1101, 2002.

[5] Cai N, Kou S G, "Option pricing under a mixed-exponential jump diffusion model", Management Science, vol. 50, no. 9, pp. 1178-1192, 2011.

[6] Hull J C, White A, "The pricing of options on assets with stochastic volatilities", Journal of Finance, vol. 42, no. 2, pp. 281-300, 1987.

[7] Heston S L, "A closed-form solution for options with stochastic volatility with applications to bond and currency options", Review of Financial Studies, vol. 6, no. 2, pp. 281-300, 1993.

[8] Duffie D, Pan J, Singleton K, "Transform analysis and asset pricing for affine jump-diffusions", Econometrica, vol. 68, no. 6, pp. 1343-1376, 2000.

[9] Vasicek O, "An equilibrium characterization of the term structure", Journal of Financial Economics, vol. 5, no. 2, pp. 177-188, 1977.

[10] Cox J C, Ingersoll J E, Ross A S A, "A theory of term structure of interest rates", Econometrica, vol. 53, no. 2, pp. 385-407, 1985.

[11] LI W H, LIU L X, LV G W, et al, "Exchange option pricing in jump-diffusion models based on esscher transform", Communications in Statistics - Theory and Methods, vol. 47, no. 19, pp. 4661-4672, 2018.

[12] KIM J H, PARK C R, "A multiscale extension of the Margrabe formula under stochastic volatility", Chaos, Solitons and Fractals: the interdisciplinary journal of Nonlinear Science, and Nonequilibrium and Complex Phenomena, vol. 97, pp. 59-65, 2017.

[13] CHEANG G, GARCES L, "Representation of exchange option prices under stochastic volatility jump-diffusion dynamics", Quantitative Finance, vol. 20, no. 2, pp. 291-310, 2020.

[14] Bates D, "Jump and stochastic volatility: Exchange rate processes implicit in Deutsche market in options", Review of Financial Studies, vol. 9, no. 1, pp. 69-107, 1996.

[15] Bakshi G, Cao C, Chen Z, "Empirical performance of alternative option pricing models", Journal of Finance, vol. 52, no. 5, pp. 2003-2049, 1997.

[16] Yang H X, Kanniainen J, "Jump and volatility dynamics for the S&P 500: Evidence for infinite-activity jumps with non-affine volatility dynamics from stock and option markets", Review of Finance, vol. 21, no. 2, pp. 811-844, 2017.

[17] Yeap C, Kwok S S, Choy S, "A flexible generalized hyperbolic option pricing model and its special cases", Journal of Financial Econometrics, vol. 16, no. 3, pp. 425-460, 2018.

[18] Duffie D, Kan R, Singeton K, "Transform analysis and asset pricing for affine jump-diffusion", Econometrica, vol. 68, no. 6, pp. 1343-1376, 2000.