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Application of Matrices in Engineering

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Abstract: ***Mathematics plays importance role in our daily life. Branch of Mathematics are as given, Real analysis, vector analysis, group theory, linear algebra, vector algebra, ordinary differential calculus, integration, discrete mathematics, Matrices and determinant etc. Throughout time, matrix have played a crucial role in solving sets of linear equations. In branch of applied mathematics & many branches of applied science matrix are used. We can check the outcome of a matrix in MATLAB to see the images created by computers. In physics, optics, which is the science of light, used matrices even before they were used in computer - generated images. In applied mathematics, matrices have diverse applications, Such as signal processing, image processing and more. For instance, in graph theory, matrices help represent connections between nodes, with the integer values in an adjacency matrix indicating the number of connections a particular node possesses.***

Keywords: Linear Algebra, Matrix theory, Determinant, lattices, Number theory, discrete mathematics

1. Introduction, Notation and Definition

Matrix is rectangular $\{a_{ij}\}$or square$[a_{ij}]$ arrangement of element in a row and column. In the matrix form we can arrange complex data in a simple form. These organised arrays provide a concise and powerful representation of complex data, images, number or object. Element are given by a_{11}, a_{12}, a_{13} ….. we can add two matrix of same order, multiply the matrix by scalars. In the field of computing, matrices are used in message encryption. They are used to create three - dimensional graphic images and realistic looking motion on a two - dimensional computer screen and also in the calculation of algorithms that create Google page rankings. Matrices are used to compress electronic information and play a role in storing fingerprint information. An m × n array we read m by n network contains m lines and n sections, while m and n are called its measurements and m $\times n$ denote the request of lattice. For example, in 3×5 grid there are 3 lines and 4 sections.

Matrices play a crucial role in cryptographic systems, contributing to both encryption and decryption processes. The application of matrices in cryptography provides a mathematical foundation for secure communication and data protection. Here is a discussion on the role of matrices in cryptographic systems and how they are utilized for encryption and decryption

2. History

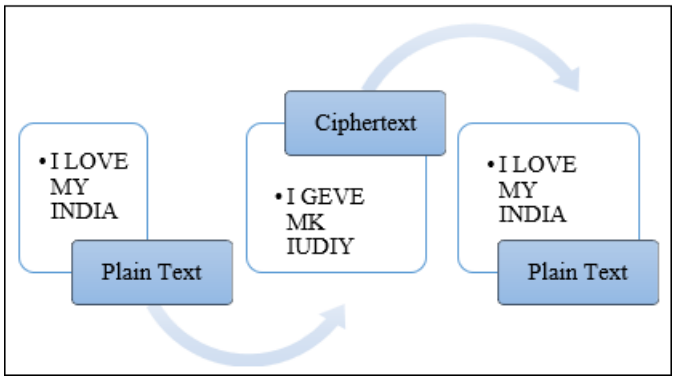
The mathematical concept of a matrix has ancient roots, with some early ideas found in Chinese mathematics. However, the formalization of matrix theory as we know it today began in the 19th century. Matrix theory and its properties were further developed by mathematicians like Arthur Cayley and William Rowan Hamilton in the mid - 19th century. Matrices play a significant role in computer science, particularly in the field of graphics and numerical analysis. They are used to represent transformations, solve linear equations, and perform various operations in computer algorithms. Matrices are fundamental in computer graphics for transformations such as translation, rotation, and scaling. Matrices have applications in physics, especially in quantum mechanics. They are used to represent physical systems, and matrix mechanics is a formulation of quantum mechanics. In summary, the term "matrix" has a diverse history, ranging from its origins in mathematics to its applications in computer science, physics, and its iconic association with a simulated reality in popular culture.

Applications of Matrix

- Applications of Matrix in scientific Calculation.
- Applications of Matrix to practical real-life problems.
- Used for solving system of linear Equation.
- Used in representing the data like the traits of people's population growth, habits.
- In study of quantum mechanics and optics.
- Used of Matrix in Conversion of electrical energy into another useful energy.
- Used of Matrix in Coding & decoding in real life.
- Used in robotics and computer graphics.
- Matrices are used in signal processing for tasks such as filtering, compression, and transformations.
- Matrices are extensively used to represent data in machine learning algorithms.

The process of encoding and decoding is also being used
1) Steganography
2) Cryptography

Cryptography:
The basic idea of cryptography is that information can be encoded using an encryption scheme and decoded by anyone who knows the scheme. In present day we have different type of encryption schemes ranging from very simple to very complex. Most of them are mathematical in nature. Today, in the time of data science and machine learning, every second, we transmit sensitive information over the Internet, including card details, personal details, bank details, loan document, and passwords for important databases. Frequently, this information is secured through encoding or encryption. The process involves using a matrix as an encoder, and its inverse serves as the decoder. modern cryptography concerns with Confidentiality, Integrity, on - repudiation Authentication also utilized in numerous applications like keeping money exchanges cards, PC passwords, and online business exchanges.



In the Encryption process:
Encryption is the process to convert the personal massage, secret massage to make it unreadable information for unauthentic
users as well as third party. this cryptographic technic i. e. Encryption technic protects sensitive data and most secure information such as credit card numbers, online banking ID & password by encoding and transforming information into unreadable cipher text. This encoded massage may only be decrypted with a key. the encoder is a matrix and the decoder is inverse of matrix. Let B be the encoding matrix, N the message matrix, and Y will be the encrypted matrix

Then, mathematically, the operation is
BN=Y

Someone has X and knows B, and wants to retrieve N, the original message. That would be the same as solving the matrix equation for N.
Pre multiply by B^{-1}.
we have, N=$B^{-1}Y$
(B is a non - singular matrix)

There are two types of encryption:
- Symmetric - key
- Asymmetric - key

Symmetric Key Encryption:
Imagine symmetric key encryption like having two secret keys, just like twins, for computers or devices talking to each other. These keys act like locks to keep the messages safe. The first encryption kid on the block was Data Encryption Standard (DES), using a 56 - bit key, but it's not super secure. Now, think of Advanced Encryption Standard (AES) as the upgraded version – it's more reliable because it can use a 128 - bit, 192 - bit, or 256 - bit key to make sure your data stays super safe.

Asymmetric - key encryption:
Asymmetric - key encryption, or public - key encryption, is like having two special keys for sending secret messages. There's a public key, like a friendly key, that's shared with other computers. It's used to lock up the message. The private key, like a secret key, stays only on your computer. It's used to unlock and read the message. Pretty Good Privacy (PGP) is a popular system that uses these keys. Encryption, or this locking and unlocking process, is crucial for making sure sensitive information gets to the right place safely and securely.

Formulation of Problem:
Let us encode the message "I LOVE MY INDIA".
Let A =1, B=2, C=3, D=4, E=5, F=6, G=8 and so on, and blank =0.

We need to translate letters into numbers. Using the above list, the message becomes: 9, 0, 12, 15, 22, 5, 0, 13, 25, 0, 9, 14, 4, 9, 1 Now we need to decide a non - singular coding matrix, which will encode the matrix.

$$A=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 5 & 2\end{pmatrix}$$

This is a 3 x 3 matrix having 3 rows & 3 columns. We can encode 3 numbers at a time. So, we have to divide the message into group of 3 numbers, consider blanks to the end if necessary.

The first group is 9, 0, 12. So that the message matrix will be of 3x1

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 5 & 2\end{pmatrix}\cdot\begin{pmatrix}9\\ 0\\ 12\end{pmatrix}=\begin{pmatrix}9\\ 0\\ 33\end{pmatrix}$$

So, the first 3 encrypted numbers are 9, 0, 33.

Next three numbers are 15, 22, 5.

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 5 & 2\end{pmatrix}\cdot\begin{pmatrix}15\\ 22\\ 05\end{pmatrix}=\begin{pmatrix}15\\ 22\\ 135\end{pmatrix}$$

The second encrypted numbers are 15, 22, 135.

And so on, encoding the entire sequence gives us 9, 0, 33, 15, 22, 135, 0, 13, 115, 0, 9, 73, 4, 9, 51. The encrypted message is “I GEVE MK IUDIY”.

Now we decode the sentence “I GEVE MK IUDIY”, by using inverse matrix of $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 5 & 2\end{pmatrix}$.

Inverse of matrix A is

$$A^{-1}=\frac{1}{2}\begin{pmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ -1 & -5 & 1\end{pmatrix}$$

Now we decode the first 9, 0, 33 using inverse of A.

$$\frac{1}{2}\begin{pmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ -1 & -5 & 1\end{pmatrix}\begin{pmatrix}9\\ 0\\ 33\end{pmatrix}=\begin{pmatrix}9\\ 0\\ 12\end{pmatrix}$$

So, the first 3 decrypted numbers are 9, 0, 12 same as original number and sentence, “I L”

Now we next three numbers 15, 22, 135 using inverse of A.

$$\frac{1}{2}\begin{pmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ -1 & -5 & 1\end{pmatrix}\begin{pmatrix}15\\ 22\\ 135\end{pmatrix}=\begin{pmatrix}15\\ 22\\ 5\end{pmatrix}$$

So, next three number are decrypted as 15, 22, 5 same as original number and sentence, “O, V, E”.

Now we decode next three numbers 0, 13, 115 using inverse of A.

$$\frac{1}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & -5 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 13 \\ 115 \end{pmatrix}=\begin{pmatrix} 0, \\ 13 \\ 25 \end{pmatrix}$$

So, next three number are decrypted as 0, 13, 15 same as original number and sentence " MY".

By doing the same process we can decrypt all the coded number.
we get the series 9, 0, 12, 15, 22, 5, 0, 13, 25, 0, 9, 14, 4, 9, 1. And the sentence "I LOVE MY INDIA".

3. Conclusion

We have to encode & decode the sentence as per our requirements for the security purpose. Matrices are the one of the most common applications of mathematics. in this method we have to use Matrix Inversion Method.

In this case we consider one sentence. Encrypt it by matrix Multiplication & decrypt it by Inverse Matrix Method. The sentence "I LOVE MY INDIA" encoded to "I GEVE MK IUDIY" by matrix A.

Now encoded sentence "I GEVE MK IUDIY" can we decode by A^{-1} and we get original sentence "I LOVE MY INDIA".

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