An Overview of Decision theory and Game Theory and Some of its Applications in Economics & Other Social Sciences

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Abstract: Decision theory and Game theory are the special branches of mathematics that are mostly applicable in social sciences, economics, and commerce as well as in engineering sciences, Biology etc. It is the study of strategy and conflict in which agent success in making choices depending on the choice of the other opponent. It was initially developed in economics to understand a large collection of economic behaviors of firms, markets and consumers. This theory has been used to attempt or to develop theories of ethical and normative behaviors as well. Applied game theory is helpful in understanding the rational behavior of economics and philosophy. The purpose of this paper is to clarify the following. First is to provide very brief overview of decision theory, game and game theory, second to discuss background and impact of game theory, third to analyze the prisoner’s dilemma game and finally to present applications of it and special attention to economics and other social sciences.

Keywords: Decision theory, Game, Game theory, Strategy, Prisoners dilemma, economics

1. Introduction

Decision Theory: A decision in general may be defined as the selection by the decision maker of an act considered to be best according to some pre designed standard from among the several available options.

Every day we as human being make many decisions and accordingly, we make some decision that can have immediate or long-term effects on our lives. Such decision as where to attend a meeting, whether to rent a house or buy, whether one’s company should accept a merger proposal or not and so on, is important decision for which we would prefer to make correct choice.

Irrespective of the type of decision-making problem there are certain essential elements which are common to all such problems. These are –

Courses of action: A decision is made from among a set of definite alternative courses of action these are also called actions, acts or strategies and are under control and known to the decision maker.

State of nature: consequences of many course of action are dependent upon certain factors beyond the control of the decision maker. These factors are called state of nature.

Uncertainty: There is indeterminateness regarding which event or outcome will occur. This uncertainty is indicated in terms of probabilities assigned to events

Payoff: Each combination of a course of action and an event is associated with a payoff, which measures the net benefit to the decision maker that accrues from a given combination of decision alternatives and events. They are also known as conditional profit value or conditional economic consequences.

The availability of partial or imperfect information about a problem leads to two types of decision making situations - (i) Decision under uncertainty and (ii) Decision under risk.

A number of criteria are there for making decision under uncertainty on the basis of the assumptions that no probability distribution is available. On the other hand in the environment of decision under risk, most decision may have to be made under condition of risk. Here more than one state of nature exists and the decision maker has sufficient information to assignment profanities to each of the states.

Game: The ‘Game’ is the object of studying Game theory. A complete set of rules describes a game. A play is an instance of the game. Most game is played for fun, entertainment and leisure. When some over reacts, we sometimes say it is just a game. Chess, checkers, volleyball, football, judo, squash and thousand other formalized contests are examples of games, in which there are two players or parties or teams. Most of these games have only one winner after the play. But there are many other situations that a different strategy is applied.

A game is an abstract, which is defined as a formal description of a strategic situation. Any strategic interaction involves two or more decision makers, each with two or more ways of acting such that the outcome depends on the strategy choices of all the players. Each player has well defined performances among the entire possible outcome, enabling corresponding utilities to be assigned. A game makes explicit the rules governing player’s interactions, the player’s feasible strategies and their performances over outcomes.

All games have three basic elements players, strategies, payoffs, so a game in the normal form consist of –

(a) A finite number of players $p_1, p_2, p_3, \ldots \ldots, p_n$

(b) A strategy set $s_i$ assigns to each players $s_1, s_2, s_3, \ldots \ldots, s_n$
Categories of Games:
Games can be distinguished according to various criteria -

(i) **According to number of players:** Usually there should be more than one player. Maximum number of players is finite. A player may be expounded as a notion or a team comprising many people, companies, associates etc. games categories as a one person, two person or n - person game (n>2).

(ii) **According to rationality of players:** A key supposition in many variants of game theory is that the players are rational. A rational player is one who always chooses action which gives the outcome he mostly prefers, given that he expects his competitors to do. There can be distinguished two extreme ways. The first one is called ‘intelligent’ player who is acting rationally. The other extreme is the player who chooses random actions.

(iii) **According to cooperation:** Games can be categorized into cooperative and non - cooperative. A game in which players are allowed to cooperate with each other on a joint strategy is called a cooperative game. For example, a cooperative game is a bargaining game between two parties in a transaction over the value of Target Company. For non - cooperative a basic assumption is that individual players can not cooperate. In this game is associated with the analysis of strategic choices.

(iv) **Normal and extensive form:** The strategic form which is also called normal form is the basic type of game situated in non-cooperative game theory. A game in a strategic form lists each player and the outcomes that result from each possible combination of choices. The extensive form which is also known as a game tree is more detailed than the strategic form of game. It is a complete description of how the game is played over time.

(v) **Zero - sum and nonzero - sum games:** Zero - sum games have the property that the sum of the payoffs to the players equal to zero. For example, poker, chess and in most sports like basketball, volley ball are zero - sum game. It is also called constant sum games. In nonzero - sum games, all the players could win or loss together. Most of the games in our real life and the business world are nonzero - sum game. For example, price wars between firms are nonzero - sum games. In nonzero - sum games players have common and conflicting interests.

**Game theory:** Game theory is the logical analysis of situations of conflict and cooperation. Game theory could be formally defined as a theory of rational decision in conflict situations. Model of such situations as they are conceived in game theory involves

1) A set of decision maker called players
2) A set of strategies available to each player courses of action which he or she may choose to follow.
3) A set of outcomes, the strategies chosen by each player determine the outcome of the game.
4) A set of payoffs accorded to each player in each of the possible outcomes.

Thus game theory is the study how players should rationally play games. Each player would like to the game to end in an outcome which gives him as large a payoff as possible. He has some control over the outcome, since his choice of strategy will influence it.

The field known as ‘Game theory’ was introduced first in the last century by mathematicians and economist as a tool to analyze both economic competitions and political conflicts. Two distinguished game theorists Robert Aumann and Oliver Hart explain the attraction in the following way.

Game theory may be viewed as a sort of umbrella or ‘unified field’ theory for the rational side of social sciences, where social is interpreted broadly to include human as well as non human players. (Computer analysis). It doesn’t use different adhoc constructs. It develops methodologies that apply in principle to all interactive situations.

The subject of game theory is situations where the result for a player doesn’t only depend on his own decision, but also on the behavior of the other players. Game theory is the theory of independent and interdependent decision making the organizations where the outcome depends on the decisions of two or more outcomes players, one of which may be nature itself and where no single decision maker has full control over the outcomes.

The concept of game theory provides a common language to formulate structure, analyze and eventually understand different strategic scenarios. Game theory is a scientific discipline that investigates conflict situations, the interaction between the agents and their decisions. To perform the analysis of critical situations game theory uses not only mathematical approaches but also an important tool in economics, commerce, political science, law, psychology, philosophy and other disciplines.

2. **Background and Impact of Game Theory**

Game theory has been widely recognized as an important tool in different fields. The development of game theory is largely broadened. The initial of game theory occurred in a letter written by James Waldegrave in 1713. In this letter Waldegrave provides a mixed strategy solution to a two person version of the card game. James Madison made what we now recognize as a game theoretic analysis of the way stated can be expected to behave under different system of taxation. The earliest example of a formal game theoretic analysis is the study of a duopoly by Antoine Cournot in1838. His publication researches into the mathematical principles of the theory of wealth presents a solution that is a restricted version of the Nash equilibrium. A formal theory of game was suggested by mathematician Emile Borel in 1921, which was furthered by the mathematician John Von Neumann in a ‘theory of parlor games’ in 1928. John Von Neumann and Oskar Morgenstern published the theory of games and economic behavior in 1944. It introduced the economic and mathematical basis for the field we now call ‘game theory’. This book contained much of the basic terminology and problem set up that is still in use recent times. Von Neumann and Morgenstern established the field that economic and social question can often be described as
mathematical models of suitable games of strategy. This gigantic work provides the method for finding mutually consistent solutions for two person zero sum games. Game theory becomes closely associated with the Rand Cooperation after the Second World War. The Rand standing for R and D Cooperation was a private company spam off from the US Air force at the end of the War and it was especially concerned with the prospects of intercontinental nuclear war free. Game theory was obviously relevant to the task and as a result the organization championed its development by hiring as consultants Von Neumann and other central figures in the development of game theory like John Nash, Duncan Luce and Howard Raiffa. In 1950 game theory developed dramatically when John Nash developed game tools and concepts for general non cooperative theory and cooperative bargaining theory. He introduced what is now called the ‘Nash equilibrium’ of a strategic game in 1951. Game theory was broadened theoretically and applied to problems of war, political science and philosophy in 1950’s and 1960’s. British philosopher R. B. Braithwaite in his book ‘Theory of games as a tool for the Moral philosopher’ was applied to philosophy in 1955. In this book Braithwaite provides how games can be used to arrive at moral ethical decisions.

As a specialized field game theory was established by great mathematician and Economist, the last quarter of the 20th century and early 2000’s game theory acknowledged special attention in 1994 with the awarding of Nobel Prize in economics to John Harsanyi, John Nash and Reinhard selten. Game theorist Thomas Schelling and Robert Aumann was awarded Nobel Prize in 2005. Schelling worked on dynamic models early examples of evolutionary game theory. Aumann for having enhanced our understanding of conflict and cooperation through game theory analysis. In 2007 Roger Myerson, together with Leonid Hurwicz and Eric Maskin was awarded the Nobel Prize in economics for having laid the foundations of mechanism design theory with game theory structure and its design. Alvin E. Roth and Lloyd S. Shapley were awarded the Nobel Prize in Economics for the theory of stable allocations and the practice of market design in 2012.

Examples of Game: ‘Analysis of prisoner’s dilemma’

Classic and probably the most discussed example of the use of Game theory in the prisoner’s dilemma. The prisoner’s dilemma is an example of two person non zero sum game in which some outcomes are preferred by both players to other outcomes. It is a subset of game theory and is simplified to give each of two parties decision resulting 2x2 i.e. 4 outcomes. In 1950 Melvin Dresher and Merrill Flood at the RAND Corporation considered game could have an equilibrium outcome which is unique but fails to be Pareto optimal.

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<th>A</th>
<th>B</th>
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<tbody>
<tr>
<td>(0, 0)</td>
<td>( - 2, 1)</td>
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<td>(1, -2)</td>
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**Figure 1.1:** Original prisoner’s dilemma A is ‘don’t confess’, B is ‘confess’

Later when presenting this example at a seminar at Stanford University Albert W. Tucker told a story to go with the game and formalized. The story of prisoner’s dilemma is as follows -

Two prisoners accused of the same crime are kept in separate cells, only a confession by one or both can lead to conviction. If neither confesses, they can be convicted of a lesser offense, five years. If one confesses and the other doesn’t the first goes free. While the other receives the full sentence, 10 years in prison. Under the circumstances is it rational to admit guilt or to deny it. In this situation both suspects offered the options -

a) If both suspects not confess, they will be arrested for one month.
b) If both suspects confess, they will be arrested for 5 years.
c) If one suspect confesses and the other doesn’t, then one who confessed will get free (0) and the other who doesn’t confess will be arrested for 10 years.

The constraints involved in this situation analysis need to be introduced here.

Therefore, let R=Reward for mutual cooperation

S=Payoff for cooperation against cooperation (payoff for C/C)

T= Temptation to defect

P= Punishment for mutual defection

The general form of prisoner’s dilemma conditions: T>R>U>S and R> (S+T) /2

The situation can be described by a pay matrix as

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<th>C</th>
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<tbody>
<tr>
<td>A</td>
<td>(R, R)</td>
<td>(S, T)</td>
</tr>
<tr>
<td>B</td>
<td>(0, 0)</td>
<td>(-2, 1)</td>
</tr>
<tr>
<td></td>
<td>(T, S)</td>
<td>(P, P)</td>
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<td></td>
<td>(1, -2)</td>
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**Figure 1.2:** prisoner’s dilemma Here C: cooperative, D: Defect

**Rational behavior and decision theory:**

Game theory is a useful and potential tool for understanding of human affairs. Game theory has been expanded as a part of a general theory of Rational Behavior. Rationality is a normative concept which indicates to what we should do to attain a given end or objective. When we are thinking of behavior involving a choice of the best means available for achieving a given end that is ‘rational behavior’. Rational behavior models are widely used in game theory. According to Von Neumann and Morgenstern ‘we wish to find the mathematically complete principles which define rational behavior’ for the participants in a social economy and to derive from them the general characteristics of the behavior. Game theory of rationality advises what agents should do in specific interactive situations, given their preferences.
Decision theory is the analysis of the human behavior which concentrates on identifying the best decision option to decision maker. Decision theory provides the rationality of decision in the light of preferences over outcomes and beliefs about the likelihood of outcomes. Game theory is clearly related to decision theory, which studies inter actions between self interested players. Game theory along with decision theory analyzes interdependent decision between rational strategic agents. The basic difference between the two is that – decision theory treats all outcomes as exogenous event, ‘moves of nature’; where as in game theory the prime source of uncertainty for an agent is the way other agents will behave.

Application of Game theory: Game theory has so many applications in most of the branches like Economics, commerce, psychology, philosophy, political science etc.

Application of game theory in economics and other social sciences: Game theory is an important tool applied in mathematical economics and business for modeling the patterns of behavior of interacting agents. According to P. A. Samuelson and W. D. Nordhaus economic life contains many situations with strategic interactions among the firms’ households, governments or others. Game theory analyzes the way that two or more parties who interact in an area such as market chosen actions or strategies that jointly affect all participants.

Economist use game theory as a tool to analyze economic competition, economic phenomena such as bargaining, mechanism design, auctions, rotting theory, experimental economics, political economy, behavioural economics etc. game theory is also applied for determining different strategies in the business world. It offers valuable tools for solving strategy problems. Many business strategies are short- or long-term plans to achieve suitable profitability. A business can often successfully position in the market with right strategy and a business will suffer in the long run with long strategy. Strategic behavior occurs regularly among executives, managers and investors in business world. They must decide to enter into new market, launch new products invest now or lose the opportunity to invest and make pricing and purchasing decisions. Game theoretic models are very potential tools for analyzing firm decisions. Game theory models forces each player to consider the actions of others when picking their strategy, in which one player may respond to the moves of his competitor. It provides significant benefit to a decision maker.

Similar to economics Game theory are also applicable in politics, psychology, philosophy etc. In politics it is widely used in political affairs which is focused on the areas of international politics, war strategy, political economy etc.

3. Conclusion

Game theory is the formal study of conflict and cooperation between intelligent rational decision makers. It has been a powerful analytical tool to help us understanding the phenomena that can be observed when decision makers interact. Game theoretic models have become increasingly sophisticated and in consequence much more powerful and

References