Exploring the Beauty of Mathematics: the Application of Rolle's Theorem in Simple Harmonic Motion and Its Value in Civic Education

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Abstract: Chinese university students are required to undertake essential courses in advanced mathematics and ideological and political theory. Higher Mathematics enhances students' abstract reasoning and problem-solving skills through topics such as calculus and linear algebra. On the other hand, Ideological and Political Theory focuses on instilling accurate ideological perspectives and political consciousness, encompassing teachings on Marxism and Mao Zedong Thought. These courses collectively contribute significantly to students' holistic development, striving to achieve the educational objective of fostering well-rounded growth and enhancing moral, intellectual, physical, aesthetic, and labor-related capabilities.

Keywords: Advanced mathematics, Civic and Political Science, Mean value theorem.

1. The Importance of Advanced Mathematics Courses in Relation to Ideology and Politics.

In the contemporary era, the fundamental societal contradiction in China has shifted to the disparity between individuals' aspirations for a high-quality lifestyle and the existing uneven and inadequate development. A crucial component of a high-quality life is the pursuit of superior education. The significance of ideological and political education within the realm of education is evident. However, with the increasing specialization within the field, the cohesion among various subjects appears to have diminished, creating the perception that ideological and political education is confined to specific courses. This has resulted in a lack of proper attention and awareness in other courses, occasionally leading to counterproductive outcomes.

The concept of "curriculum ideological and political" has been implemented in select colleges and universities, resulting in notable achievements. Curriculum ideology has emerged as a crucial focus of ideological and political education in the contemporary era. Therefore, comprehensively understanding the essence, attributes, and challenges of curriculum ideology and politics, and actively seeking resolutions, are of significant importance in fully leveraging the role of ideological and political education across all types of curricula. This endeavor aims to align curriculum ideology and politics with ideological and political curricula, nurture the holistic development of human resources, meet societal expectations for an improved quality of life, and contribute to the realization of the Chinese nation's great rejuvenation.

2. Examination of the Viability of Integrating Civics into Advanced Mathematics Curricula

Rolle's theorem, a fundamental concept in calculus, holds significant importance within the realm of mathematical theory and demonstrates practical applications in physics, particularly in the analysis of simple harmonic motion. This paper aims to delve into the intricate relationship between Rolle's theorem and simple harmonic motion, highlighting the interdisciplinary nature of these concepts and showcasing the inherent beauty of mathematics. By exploring this connection, the study seeks to underscore the unique role and significance of mathematics in ideological and educational contexts within colleges and universities.

The research methodology employed in this study involves a case study approach focused on addressing specific issues. Drawing upon advanced mathematical principles and incorporating problem-solving techniques, guided instruction, and direct teaching methods, the teaching design aims to engage students actively in classroom learning and foster an appreciation for the elegance of mathematics. Additionally, by narrating the stories of renowned mathematicians and instilling their spirit of inquiry, the study aims to cultivate students' scientific reasoning and professionalism, thereby promoting cultural self-assurance and a sense of academic identity.

3. Current Challenges in Integrating Civics into Advanced Mathematics Curricula

3.1 Absence of a System to Enhance the Structuring of Civics Education within the Academic Curriculum.

Higher mathematics courses play a crucial role in university education, serving as a fundamental subject within the field of mathematics and providing essential theoretical foundations for professional studies. The significance of ideological and political education in promoting the comprehensive development of college students is evident. Consequently, there is a pressing need to seamlessly incorporate ideological and political education into higher mathematics courses to advance teaching methodologies. However, a notable challenge in this reform effort is the imperfect organizational structure of civic and political education within the course framework. Given the profound theoretical basis and central position of higher mathematics courses, the emphasis tends to be on disseminating knowledge, potentially overshadowing the integration of civic-political education. This, in turn, hinders the progress of course reform in this area. Addressing these issues and actively seeking solutions are imperative to propel the reform and advancement of higher mathematics courses.

3.2 Inadequate Provisions for the Instruction of Civics within the Educational Curriculum.

The instruction of advanced mathematics courses involves interdisciplinary collaboration. To effectively advance the reform of such instruction, it is imperative to enhance reform initiatives and augment teaching resources at the level of advanced mathematics teaching materials and aids. Despite this, there is a shortage of teaching materials for the ideological and political aspects of restructuring advanced mathematics courses. Furthermore, a significant number of advanced mathematics educators lack the essential resources to explore the ideological and political content within textbooks. They primarily focus on mathematical theories and knowledge. Consequently, there is a limited understanding of the professional knowledge related to ideological and political education, which hinders the thorough exploration of the ideological and political implications underlying advanced mathematics knowledge. The lack of resources hinders the availability of a wide range of teaching materials for enhancing ideological and political education in advanced mathematics courses, thereby impeding the smooth execution of such education.

3.3 The Teaching Approaches Utilized in the Civics Curriculum are Considered to be Antiquated.

Conventional pedagogical approaches in higher mathematics education often prioritize the impartation of theoretical knowledge and technical skills, with less emphasis placed on fostering students' ideological development and social responsibility. At times, there is an overemphasis on academic achievement, leading to a predominant reliance on lecture-based instructional methods. Furthermore, some higher mathematics instructors tend to focus excessively on the subject matter itself, overlooking the importance of providing guidance to students in terms of career planning and the development of humanistic qualities. This oversight poses challenges to the advancement of pedagogical reforms within the Civics curriculum.

4. Fundamental Methods for Instruction and Education

During the lesson, the instructor adeptly incorporates the concept of curriculum thinking with Rolle's theorem, constructs mathematical scenarios, encourages students to identify issues and pose inquiries stemming from these scenarios, and focuses instruction on the fundamental principles of resolving these foundational problems. Novices often encounter challenges in grasping the rich historical context and concepts underpinning the median theorem, as they are immediately presented with concepts, theorems, properties, formulas, and exercises as the crux of calculus. This overly pragmatic pedagogical approach can deter inquisitive beginners, leading to a prevailing sentiment of disengagement with the median theorem.

To address this issue, educators delve into the underlying political dimensions of advanced mathematics courses and transform them into potent educational resources through meticulous instructional design, with the goal of fostering students' holistic development. Specifically, it is posited that instructors can commence with the advanced mathematics textbook and link Rolle's theorem with simple harmonic motion in high school physics. Through such interdisciplinary integration, students can deepen their comprehension of the practicality and significance of the mathematical theorem, while also nurturing their interdisciplinary thinking skills and innovative mindset.

4.1 Utilizing Mathematical Principles to Provide Guidance to Students in Their Academic Pursuits.

In the introduction of Rolle's theorem, educators can employ a range of instructional methods, in conjunction with course content, to narrate a compelling mathematical account that conveys the notion that knowledge and historical developments are the result of human endeavor. This approach aims to imbue students with the perception that the study of advanced mathematics involves an interactive process of learning and living. It is essential for higher mathematics courses to be engaging and dynamic, fostering an environment where learning is not only active but also imbued with vitality. By tracing the historical evolution of limits, students can appreciate the concealed elegance, profound intricacy, and abstract allure of mathematics, thereby motivating them to cultivate a mindset of perseverance and aspiration towards achievement.

4.2 Fostering Students' Inclination Towards Synthesizing Information Throughout the Educational Experience.

The process of acquiring advanced mathematical knowledge differs from that of basic mathematics by being segmented into three distinct stages: initial comprehension, accurate recall, and adaptable integration. As students engage with new mathematical concepts, formulas, or theorems, their understanding typically remains superficial. For instance, linking Rolle's theorem with the familiar concept of simple harmonic motion from high school can enhance students' comprehension of the underlying significance conveyed by Rolle's theorem.

According to the geometric interpretation of Rolle's Theorem, when considering the continuous curve represented by y=f(x) on the interval [a,b], if the arc AB formed by this curve has tangents that are not perpendicular to the x-axis at all points except the endpoints, and the x-coordinates of the endpoints A and B are equal, then there exists at least one point C on the arc AB where the tangent to the curve is parallel to the x-axis.

Rolle's theorem states that for a function f(x) defined on the

real numbers, if it meets the following criteria:



Figure 1: Diagram summarizing Rolle's theorem

(1) it is continuous on the closed interval [a,b], (2) it is differentiable in the open interval (a,b), and (3) f(a) is equal to f(b), then there exists at least one point ξ within the interval (a,b) where the derivative of the function f(ξ) equals zero.

Simple Harmonic Motion is defined as a type of vibration where the force acting on an object is directly proportional to its displacement and acts in the opposite direction.



The amplitude denoted by A, angular velocity denoted by ω , initial phase denoted by φ , and time denoted by t are key variables in the context of simple harmonic motion. The utilization of Rolle's theorem in this context primarily involves identifying critical points within the oscillatory process. This can be achieved by examining the zero crossings of the acceleration function $a=-\omega^{2*}x(t)$ and determining the extrema of the velocity function.

Engagement with Rolle's theorem enables students to recognize that comprehension is a progressive process, transitioning from intuitive awareness to logical understanding, and subsequently applying this understanding to inform practical applications. This iterative process of "practice - understanding - practice - understanding" mirrors a cyclical pattern akin to a wave rising and falling. Such an approach cultivates students' capacity for dialectical thinking and imparts the importance of establishing connections in the pursuit of advanced mathematical knowledge.

4.3 Strengthening Students' Sense of national Pride and Cultural Confidence can be Achieved by Exploring the Historical Evolution of the Median Theorem.

China's earliest mathematical treatise, "Nine Chapters of Arithmetic," documents the remarkable advancements in ancient mathematics, surpassing other countries by over 1800 years in terms of scholarly contributions. The Zhou Thigh Calculus contains significant mathematical theorems such as the Gou Shu Theorem, the "Zu Rate," Yang Hui Triangle, and the Remainder Theorem, which were developed hundreds or even thousands of years ahead of other nations. Despite the isolation imposed by the feudal dynasty hindering China's participation in the industrial revolution, the country managed to catch up over two centuries after the introduction of the median theorem. The exploration of mathematical problems has played a pivotal role in propelling the rapid advancement of information technology in China, encompassing areas such as quantum information technology, material processing, 5G technology, and logistics and distribution chains. Moreover, esteemed Chinese mathematicians from previous generations, including Hua Luogeng, Chen Shengshen, Chen Jingrun, and Su Buging, have achieved remarkable feats by overcoming challenges in adverse circumstances. Educators can leverage these historical mathematical materials to inspire individuals to uphold truth, foster independent thinking, and aspire to contribute meaningfully to their country and society.

4.4 Improving Students' Understanding of Social Responsibility Through Hands-on Activities.

Educators are encouraged to provide students with a diverse array of educational resources, foster independent learning habits, promote collaborative summarization of acquired knowledge, and facilitate the demonstration of learning outcomes through written reports or presentations. It is recommended to refrain from directly providing correct answers to students, but rather guide them towards self-discovery through hands-on activities and exploration, thereby leveraging the unique attributes of each learner. This approach not only cultivates students' abilities in generalization, summarization, problem analysis, and resolution, but also aids in their mastery of subject matter.

In 1959, the renowned mathematician Hua Luogeng emphasized the pervasive influence of mathematics in various facets of existence, spanning from the vast expanse of the universe to the infinitesimal realm of particles, and from the velocity of rockets to the ingenuity of the chemical industry. The dynamic nature of the Earth and the enigmatic essence of living organisms are intricately intertwined with mathematical applications. Mathematical modeling serves as a potent tool in enabling students to employ their mathematical knowledge for practical issue resolution, thereby translating theoretical learning into tangible real-world applications.

5. Conclusion

The Curriculum Civics initiative represents not only a reform in educational curriculum, but also a revolutionary approach that elevates traditional teaching methods. In the context of advanced mathematics courses, it is imperative to delve deeply into the scientific principles embedded within the curriculum content and cultivate students' determination to pursue genuine knowledge, innovation, and entrepreneurial skills. A genuine passion for education, coupled with a strong sense of responsibility and a mission to educate citizens for the betterment of the nation, is essential for individuals to actively engage in self-improvement throughout the teaching and educational process, incorporating principles of curriculum and political ideology. Curriculum Civics is a comprehensive, long-term endeavor that requires ongoing dedication to achieve excellence.

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