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A Review of Research on Mathematical Modeling Problems and Self-efficacy of Middle School Students

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Abstract: ***Introducing mathematical modeling problems into middle school mathematics classes is of great significance for cultivating middle school students' awareness and ability of mathematical innovation and mathematical application. This study uses the method of literature analysis to review the current research in related fields and finds that middle school students generally have cognitive barriers in the process of solving mathematical modeling problems. The seven-step cycle mathematical modeling schema (SSCS) can be used to analyze students' cognitive barriers in the modeling process. In addition, existing studies combining middle school mathematical modeling with self-efficacy use the self-efficacy scale and find that students often show low self-efficacy when facing mathematical modeling problems, which indicates that middle school students have emotional barriers when facing mathematical modeling problems. Finally, some studies have found that improving the teaching of mathematical modeling in middle schools can help students eliminate some of these barriers.***

Keywords: Mathematical Modeling Problems, Mathematical Modeling Schema, Middle School Mathematics Teaching, Self-Efficacy**.**

1. Introduction

Introducing mathematical modeling problems in middle school mathematics education can help improve students' ability to apply and innovate mathematics, but from the current teaching status, there is a lack of introduction of such problems in middle school mathematics. Studies have shown that the obstacles that students encounter in the process of mathematical reasoning may be caused by two aspects. First, they may come from the cognitive aspects of students; second, there is an interaction between cognition and emotion in mathematical activities (Barnes, 2019; DiMartino & Zan, 2011). In addition, Hannula (2012) believes that emotions can focus and bias cognition, so difficulties may be related to students' emotions in the process of mathematical reasoning. So, for mathematical modeling problems that are more difficult for students in middle schools, whether students also have obstacles in these two aspects and what efforts can be made to eliminate students' obstacles in the process of mathematical modeling are one of the issues of great concern in mathematics education today.

2. Research Questions

Based on the research on mathematical modeling for middle school students, this paper raises the following questions:

1) Do the obstacles to mathematical modeling among middle school students also manifest themselves in cognitive and emotional aspects?

2) How to analyze the obstacles that middle school students face in mathematical modeling?

3) Are there any methods to help middle school students improve their self-efficacy and mathematical modeling ability?

3. Research Methods

The study adopts the method of literature analysis to review papers on the topics of "mathematical modeling education", "mathematical self-efficacy", "mathematical modeling in middle schools", etc. All papers in this article are from Springer, Taylor Francis Online, EBSCO, which provide access to many scientific literatures from journals, books, series, protocols, reference works and conference proceedings in different disciplines and sub-disciplines through different languages.

4. A Review of Research on Mathematical Modeling Problems

4.1 Definition and Characteristics

Mathematical modeling is a problem based on the real world, which has great uncertainty. It requires the test taker to build a mathematical model based on the actual problem, solve the mathematical model, and then solve the actual problem based on the result. The mathematical model established in this process is an idealized, simplified, and formalized expression of a part of the real world. It can be divided into a ternary combination (D, M, f), namely the real world D, the subset of the mathematical world M, and the mapping f from D to M (Niss et al., 2007).

Mathematical modeling problems are different from the problems we see in daily classes or exams. Most of the problems we currently encounter in the process of mathematics learning in school education are closed problems, and openness is the most distinctive feature of mathematical modeling problems. The openness of mathematical modeling can be described by three characteristics: initial state, intermediate state, and target state (Schukajlow & Krawitz, 2023). Some scholars have compared mathematical modeling problems with application problems and pure mathematical problems (Krawitz & Schukajlow, 2017). The most significant difference between mathematical modeling problems and application problems is that application problems do not require structuring and idealization of given information, nor do they require careful interpretation and verification of mathematical results based on real-world situations (Krawitz et al., 2016). The contrast between mathematical modeling problems and pure mathematical problems is even more obvious: mathematical modeling problems require an understanding of the world outside of mathematics.

4.2 SSCS Mathematical Modeling Schema

Although many scholars have proposed schemas for mathematical modeling, there is a seven-step model that combines linguistics, applied mathematics, and cognitive psychology, namely SSCS. In most cases, SSCS seems to be very useful (Blum, W. 2015). It means that when we ideally build a mathematical model to solve practical problems, we usually go through seven steps, which are not linear but circular, forming a so-called modeling cycle (Blum & Leiß, 2007a).

These seven steps include: understanding a situation and building a mental model of it, building and simplifying the mental model, mathematizing the resulting real model, working within the mathematical model, interpreting the mathematical results, verifying these results, and proposing solutions (Durandt & Blum, 2022). It is not difficult to find from Figure 1 that the modeling process is divided into two parts, one is the real world, including steps 1, 2, 6, and 7; the other is the mathematical world, including step 4; steps 3 and 5 serve as a bridge between the two worlds.

With the development of computer technology, people have found that digital technology can be used as a powerful tool for modeling activities, not just in the internal stage of mathematics (eg, Borba and Villarreal 2005; Henn 2007; Geiger 2011; Greefrath et al. 2011). Computers can be used for experiments, investigations, simulations, visualizations, or calculations. Greefrath suggested extending the modeling cycle by adding a third world, the technical world, as shown in Figure 2.

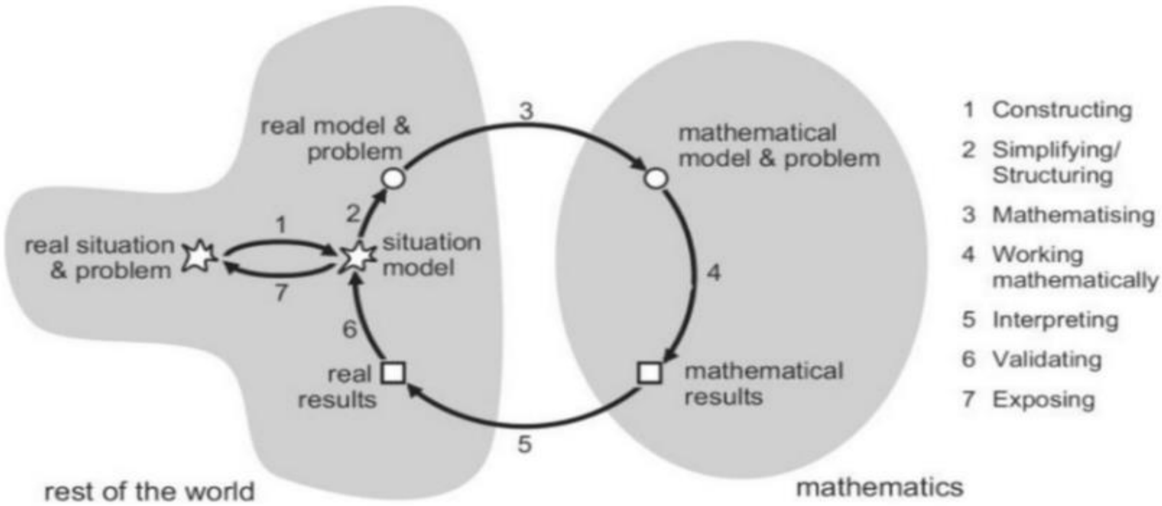

Figure 1: SSCS mathematical modeling 7-step cycle diagram

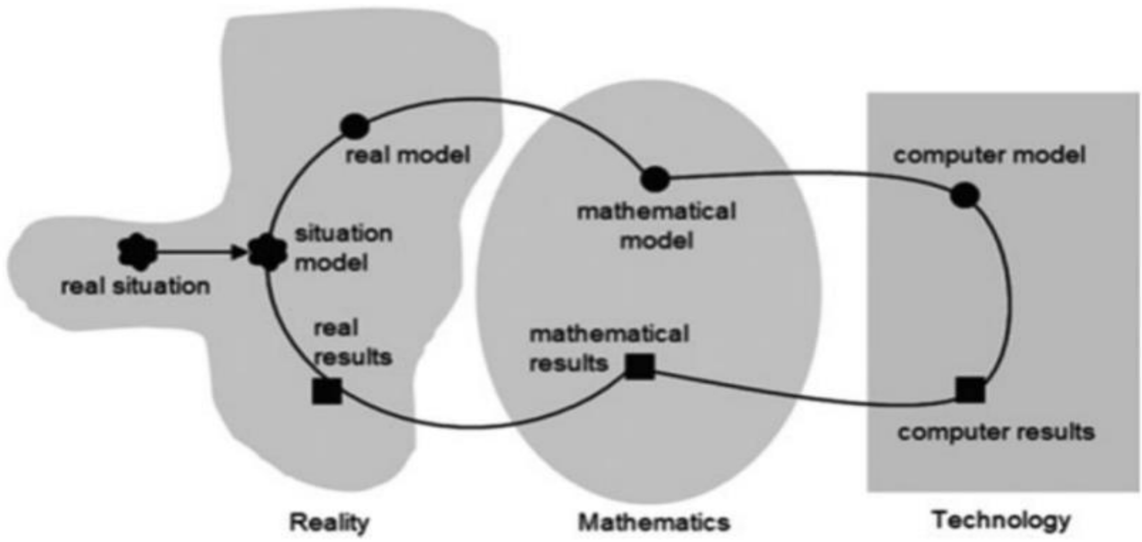

Figure 2: SSCS after joining the technology world

4.3 Significance to Mathematics Education in Middle Schools

Several studies on the development of mathematical modeling skills among middle school students have shown that each step in the modeling process (see Figure 1) is a potential cognitive barrier for students (Goos et al. 2002; Galbraith and Stillman 2006; Stillman 2011). "The weakest link in their modeling chain limits their ability" (Treilibs et al. 1980). A study analyzed students' cognitive barriers when solving open-ended modeling problems and evaluated the impact of teaching prompts on their success in solving such problems (Stanislaw Schukajlow et al, 2023), which proves that the seven-step cyclic mathematical modeling diagram has strong practical significance for the teaching and learning of mathematical modeling.

What is the significance of introducing mathematical modeling into daily mathematics teaching in middle schools? Some researchers have listed four groups of reasons (see, eg, Blum and Niss 1991; Blum 2011):

- The "pragmatic" argument: In order to understand and master real-world situations, appropriate applications and modeling examples must be treated explicitly; we cannot expect any transfer from mathematical internal activities.
- "Formative" argumentation: Competence can be improved by engaging in modelling activities, especially as modelling skills can only be improved in this way, whereas argumentation skills can be improved through "reality-related demonstrations" (Blum, 1998).
- The "cultural" argument: Relationships with the world outside mathematics are indispensable for a full picture of mathematics as a science.
- "Psychological" arguments: Real-world examples may help increase students' interest in mathematics, prompt students to construct mathematical content on their own, and help students understand mathematics better and remember it longer.

5. A Review of Research on Self-efficacy

5.1 Meaning and Application

According to Bandura's theory, self-efficacy is a good measure of a person's self-evaluation (cf. Bandura, 1977, 1986). Later, he expressed self-efficacy more clearly as "people's judgments of their own ability to organize and execute the courses of action required to achieve a specified type of performance" (Bandura, 1986, p. 391).

Mathematical self-efficacy can be seen as an important part of students' mathematical belief system (cf. Philipp, 2007, for the general concept of beliefs and Törner, 2015, for current developments). Studies have found that task-specific mathematical self-efficacy can even predict students' future career choices better than self-concept and test scores (Hackett & Betz, 1989).

make judgments by interpreting and evaluating their previous experiences (Usher and Pajares 2009; Bandura 1997). If they have gained an experience in the past, they are more likely to believe that they will also perform well when solving similar problems in the future. Especially for secondary school mathematics, mastery experience has been found to be the most powerful source of self-efficacy beliefs. As a result, students' mathematics self-efficacy is becoming a hot topic in empirical research in mathematics education.

5.2 Measurement Method

In the past, some scales have been proven to be effective and used to measure students' self-efficacy in mathematics. Betz and Hackett created the Mathematics Self-Efficacy Scale (MSES). In 2003 and 2012, the PISA assessment framework used a scale to measure mathematics self-efficacy. In addition, many scholars have created a 5- Likert scale based on the operational definition of mathematics self-efficacy to measure students' self-efficacy in specific mathematics tasks.

The measurement method of mathematics self-efficacy has two areas: one is the area of mathematics self-efficacy directly related to academic performance, and the other is the area of mathematics self-efficacy related to other factors. (Wenhua Yu & Shuodi Zhou, 2022). Detailed analysis of measurement tools shows that task specificity is often established by adding references to math problems in self-report items without showing the actual math tasks (Stanislaw Schukajlow & Dominik Leiss et al., 2011). A typical example is "I feel that it is important for me to be good at solving problems involving math or mathematical reasoning (ranked from not at all important to very important)" for perceived task value (Eccles & Wigfield, 1995) or the question "How much do you enjoy doing math-related tasks at school?" (Nurmi & Aunola, 2005). To date, math tasks have rarely been used to measure emotions, attitudes, and beliefs (Betz & Hackett, 1983). Constructing task-specific instruments to measure emotions in math could be an important step in math education.

6. A Review of Teaching Research on Mathematical Modeling

6.1 Middle School Students' Self-efficacy in Facing Mathematical Modeling Problems

Janina Krawitz and Stanislaw Schukajlow compared students' task value and self-efficacy in modeling, application problems, and internal mathematical problems through experiments, and concluded that students' self-efficacy and task value were the lowest in mathematical modeling problems (Krawitz, J., Schukajlow, S, 2018).

6.2 The Effect of Teaching Methods on Middle School Students' Mathematical Modeling Ability and Self-efficacy

Stanislaw Schukajlow, Werner Blum and others found through empirical research that improving the teaching of modeling problems had a positive impact on some of the students tested, and that student-centered teaching methods produced the most favorable effects (Schukajlow, S., Leiss, D., Pekrun, R. et al., 2015). Some studies conducted experiments based on problems in real situations and pointed out that students in unfavorable situations had low self-efficacy before. Teaching students to construct multiple solutions during the learning process can further improve students' self-efficacy (Schukajlow, S., Achmetli, K. & Rakoczy, K, 2019).

6.3 Factors Affecting Mathematical Modeling Teaching

Teachers with high pedagogical content knowledge use tasks with high cognitive activation potential and provide good support for individual student learning (Kunter & Baumert, 2013). Therefore, it seems very important to successfully impart pedagogical content knowledge of a specific subject. Some scholars have studied the subject knowledge of mathematical modeling teaching by pre-service secondary school teachers, emphasizing the importance of mathematical modeling teaching workshops for teachers to impart mathematical modeling knowledge (Greefrath, G., Siller, HS., Klock, H. et al., 2022). Previous researchers have divided the dimensions of ability related to mathematical modeling teaching content and created a scale based on this, which can be used as a reference for future research (Borromeo Ferri, 2018; Borromeo Ferri & Blum, 2010).

Research evaluating the effects of prompting multiple solutions versus prompting a single solution in mathematics instruction showed that prompting students to find multiple solutions did not directly improve their performance, but they found an indirect effect on students' performance through the number of solutions they developed and their experience of competence (Schukajlow, S., Krug, A. & Rakoczy, K, 2015).

In addition, some empirical evidence suggests the importance of mathematical content on modeling ability, such as the intervention study by Schukajlow et al., which found significant differences in the development of students' modeling outcomes on problems involving the Pythagorean theorem and linear functions (Schukajlow et al. 2015a). These findings suggest that mathematical content plays an important role in modeling. In addition, these findings can be viewed as indicating that mathematical content also influences students' task value and self-efficacy expectations (Wigfield and Eccles 2000; Eccles and Wigfield 1995).

7. Discuss

This study is based on the two major obstacles in students' mathematics learning process - cognitive obstacles and emotional obstacles, and analyzes the mathematical modeling ability and mathematical self-efficacy of middle school students in the process of mathematical modeling. Through previous research in the field of mathematical modeling and self-efficacy, the student's mathematical modeling process schema (SSCS) is elaborated in detail, some empirical research results are obtained, and suggestions for improving students' mathematical modeling ability and self-efficacy are summarized.

By reviewing the previous research in the field of mathematical modeling, we concluded that the definition of mathematical modeling is a process that requires respondents to establish mathematical models based on practical problems, solve mathematical models, and then solve practical problems based on the results; its greatest characteristics are openness and uncertainty. In addition, we found a widely used mathematical modeling process diagram (SSCS), which is conducive to our more detailed analysis of students' mathematical modeling process in subsequent research. Based on this, we analyzed the mathematical modeling teaching reform path to improve students' mathematical modeling ability and the significance of introducing mathematical modeling into teaching.

By reviewing previous research in the field of mathematical self-efficacy, we have a clearer understanding of the meaning of mathematical self-efficacy and its significance to students in the process of mathematical learning. We have also compiled some measurement methods for mathematical self-efficacy, which will be helpful for future research.

Research shows that students' mathematical modeling performance is strongly correlated with their self-efficacy in this specific task. Compared with other types of mathematical problems (i.e., application problems that often appear on mathematics papers and purely internal mathematical problems), middle school students have the worst self-efficacy for mathematical modeling problems. This low self-efficacy is largely due to the openness and uncertainty of mathematical modeling problems themselves. At the same time, we also speculate that this phenomenon may be caused by the lack of introduction of mathematical modeling problems in daily teaching. And studies have shown that a student-oriented teaching style is more conducive to students' mathematical modeling ability, task value and self-efficacy. In addition, through the analysis of the influencing factors in the process of mathematical modeling learning, it is found that the knowledge mastery of middle school teachers in the field of mathematical modeling, the training of students in multi-solution tasks, and the mastery of students in the field of mathematical content have a certain impact on the teaching and learning of mathematical modeling.

Therefore, there is a reasonable reason to believe that students can improve their mathematical modeling ability and self-efficacy in specific mathematical modeling tasks through improved teaching. It should be noted that the improvement of teaching must be based on a full understanding of students. In this process, understanding the mathematical modeling process schema (SSCS) may be of great help.

In general, the introduction of mathematical modeling in middle school mathematics is a hot topic in middle school mathematics teaching. However, the impact of the introduction of mathematical modeling in middle school on middle school students' self-efficacy is still a new area that needs a lot of exploration, which is reflected in the few research papers on mathematical modeling self-efficacy. Therefore, one of the significances of this study is to point out the direction for future research in this field.

8. Conclusion

Most of the research in the field of mathematical modeling uses the seven-step mathematical modeling process schema

(SSCS) as an explanation of the path for students to solve mathematical modeling problems, which helps to analyze students' cognitive barriers in this process. A few studies have focused on students' self-efficacy in solving mathematical modeling tasks and proposed some methods to measure students' mathematical self-efficacy.

Through the existing empirical research results, we can find that students have two obstacles in mathematical modeling: cognitive obstacles are manifested in that students often need intervention in the problem-solving process to successfully solve the problem, and students cannot cope well with the openness and uncertainty characteristics of mathematical modeling problems; emotional obstacles are manifested in that students often show low self-efficacy when facing mathematical modeling problems.

Introducing mathematical modeling in middle schools has profound significance. Existing experimental results have confirmed that changing teachers' teaching design, adopting student-oriented teaching methods, and conducting targeted training for students in this area can help students improve their mathematical modeling ability and self-efficacy in this task.

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