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Practice and Research on Diversified Problem Solving Strategies in High School Functions

Gang Xiao*, Yuqing Lan

School of Mathematics and Physics, Yibin University, Yibin, Sichuan, China
Correspondence Author

Abstract: ***As the new curriculum reform advances, high school mathematics teaching is gradually shifting from a single problem-solving model to the cultivation of diversified thinking. Functions, as a core component of mathematics, not only influence students' performance but also play a crucial role in enhancing logical thinking and innovative abilities. This article, based on teaching practice and case analysis, explores the application of divergent thinking, reverse thinking, and the integration of numbers and shapes in solving function problems. The aim is to build a flexible knowledge network for students and improve their comprehensive mathematical literacy.***

Keywords: High school mathematics, Function problem solving, Thinking innovation, Strategy diversification**.**

1. Foreword

In recent years, the mathematical proposition of the college entrance examination gradually pays attention to the breadth and depth of students' thinking, and the traditional "template" problem solving mode has been difficult to deal with complex questions. Function problems have become one of the main difficulties for students because of their abstraction and variability. How to break through the inherent thinking pattern in teaching and guide students to analyze problems from multiple angles has become the focus of educators. By integrating teaching cases, this paper puts forward diversified problem-solving strategies to provide new ideas for function teaching in senior high school.

2. Current Situation and Challenge of High School Function Problem Solving

At present, most students have the following problems in function learning:

1) Knowledge fragmentation: memorizing formulas while ignoring their geometric meaning, such as overlooking the role of even function symmetry in graph analysis. For example, when analyzing properties of even functions $f(x) = x^4 - 2x^2 + 1$, merely reciting the conclusion "symmetric about the y-axis" fails to quickly find the extremum points through graphical symmetry.

2) Thinking limitation: relying on a single solution path, lack of flexibility in the face of complex problems $f(x) = x^3 - 3x^2 + 4$. Case study: To find the extreme value of the function.

General solution method: the derivative method. For the derivative, the derivative is zero critical point and substitute the original function to calculate the extreme value.

Higher order solution: using polynomial factorization. $f(x) = (x-1)^3 + 3(x-1)$

Combination of numbers and shapes: draw the function image, observe the inflection point and the extreme value position.

Analysis: Most students only use the derivative method to lack flexibility, ignore the auxiliary role of factorization or image method, resulting in low efficiency of problem solving, especially in the absence of computing tools is difficult to verify the results.

3) Weak application ability: unable to migrate the function model to practical problems, such as using trigonometric functions to solve measurement problems; for example $C(x) = ax + \frac{b}{x}$, when solving the "optimal inventory cost" problem, it is difficult to combine the cost function with mean inequality or derivative method, resulting in modeling failure.

The above problems reflect that the teaching method of simply pursuing the correctness of the answer can no longer meet the needs of ability cultivation, and it is urgent to introduce diversified strategies.

3. The Practical Path of Diversified Problem-solving Strategies

3.1 Divergent Thinking: One Problem More Solutions to Expand the Breadth of Thinking

Case 1: To find the minimum value of the function, ().

Iquality method: x + 2 (mean inequality) $f(x) = \frac{4}{x} \geq \sqrt{x\frac{4}{x}} = 4$

Derivative method: obtain the derivative $f'(x) = 1\frac{4}{x^2}x = 2$, the critical point, and substitute into the minimum value of 4.

Parameter substitution: Let, and convert $t = xf(t) = t + \frac{4}{t}$ it to. Solve it by using the properties of the inverse function.

Geometrical meaning: analyze the $x > 0$ "U" curve of the function image, and determine the extreme value combined with symmetry.

Advantages: Cultivate the ability to analyze problems from multiple perspectives and avoid rigid thinking. Help students understand the interrelation between different mathematical tools (algebra, calculus, geometry). Enhance problem-solving strategy awareness by comparing methods of efficiency (such as completing the square being faster and derivatives being more versatile).

Reflection and improvement: the applicability is different, the derivative method is more effective for non-quadratic functions, and students need to be guided to distinguish the best application scenarios of the method. Students' cognitive load, some students may be confused due to too many methods, and they need to train in stages. First, master the basic methods (such as matching methods), and then gradually introduce higher-order tools (such as derivatives). Deep understanding is needed, emphasizing the essential connection of different methods (such as the critical point of the derivative method corresponds to the vertex of the matching method), to avoid mechanical memory [1].

3.2 Reverse Thinking: Reverse Push the Essence of the Problem

Case 2: The range of the known function is. Find the $f(x) = ln(kx^2 + 1)$ $[0, +\infty)$ range of the parameter k.

Reverse analysis: the value domain is, i.e., the launch. Combining $[0, +\infty)$ $ln(kx^2 + 1) \geq 0kx^2 + 1 \geq 1kx^2 \geq 0$ domain R, k> 0.

Positive verification: if, it $k \leq 0kx^2$ may be negative, resulting in the definition domain is not satisfied

Advantages: Strengthen the understanding of the function definition domain and the value domain relationship, and improve the ability of logical reasoning. Break the inertia of "positive derivation", and cultivate the habit of problem disassembly and reverse verification.

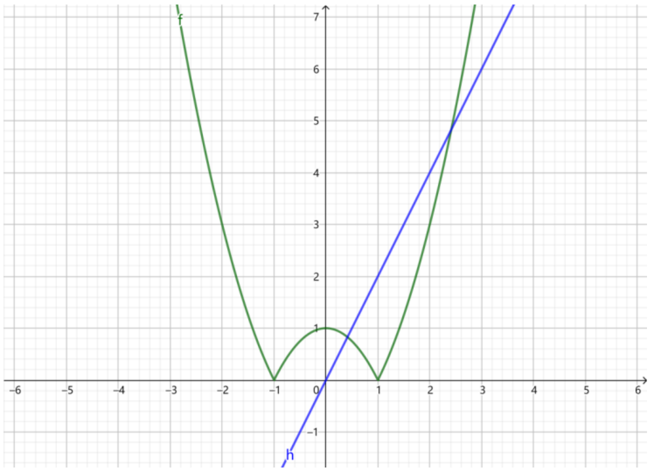
Reflection and improvement: With abstract challenges for students, some students may be difficult to reverse the conditions from the results, and they need to be guided step $x^2 + 1 \geq 1$ by step with specific examples (such as parameter separation). Students are easy to ignore the implied conditions (such as), and they need to strengthen the sense of details by comparing the wrong answers. Emphasize that the applicable scenario is more suitable for domain, domain of value or existence problems, and the scope of application needs to be defined to avoid over-generalization [2].

3.3 Combination of Numbers and Shapes: Visualization-assisted Decision Making

Case 3: $Solve\ the\ equation \mid x^2 - 1 \mid= 2x$

Algebraic method: discuss and, solve or. $x^2 - 1 = 2xx^2 - 1 = -2xx = 1 + \sqrt{2}x = -1$

Image method: draw and image, observe $y = \mid x^2 - 1 \mid$ $y = 2x$ the intersection abscissa.



Advantages: Intuitively display the range of solutions, reduce the threshold of abstract thinking (such as the "V" shape image of the absolute value function). Enhance geometric intuition and help students understand the practical meaning of algebraic solutions.

Reflection and improvement: In terms of drawing accuracy, students may cause solution set errors due to drawing error, so they need to draw [3] together with the training specifications of tools (such as GeoGebra). The limitation of complex functions, for high degree, segmented or dynamic functions, the image method may not be efficient enough, and needs to be used complementary with the algebraic method. Apply different scenarios, preferentially for simple functions or validation answers, over completely alternative algebraic derivation.

4. Suggestions on Teaching Strategy Optimization

1) Stratified teaching design: to for the basic differences. For example, focus on image analysis for weak students, and introduce parameter discussion for healthy students.

2) Interdisciplinary integration: combining physical and economic cases, such as using exponential functions to simulate population growth, to enhance application awareness.

3) Technology empowerment: Use programming to simulate the dynamic changes of functions (such as Python drawing) to deepen the understanding of properties such as extremum and monotonicity of functions.

5. Conclusion

1) Students' perspective: Diversified problem-solving strategies compare different methods to help students establish a knowledge network and reduce thinking patterns.

2) Teaching perspective: Teachers should shift from "answer-oriented" to "process-oriented", and pay attention to the penetration of methodology and the correlation between tools.

3) Proposition perspective: The college entrance examination questions gradually dilute the “routine” questions, emphasizing the ability of multi-step analysis and strategy selection, and diversified training is imperative.

4) Future outlook: AI technology (such as intelligent question bank) can be explored to recommend personalized problem solving paths for students to achieve accurate improvement. The core of the diversified problem-solving strategy is to cultivate the students’ thinking flexibility. Through multi-dimensional training, students can not only cope with the exam effectively, but also form innovative consciousness and critical thinking [4] in the exploration. Future research can further explore the application of artificial intelligence-assisted teaching in the recommendation of personalized problem solving path, and promote the development of mathematics education to a higher level.

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