Research on the Solution of Solid Geometry Problems in College Entrance Examination Mathematics — Take the National I/II/III Test from 2018 to 2022 as an Example

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Abstract: *As the college entrance examination reform progresses, the difficulty of test questions has also increased. Solid geometry, as one of the key points in the mathematics section of the college entrance examination, occupies a significant portion and is challenging. This paper, based on the national mathematics papers from 2018 to 2022, employs literature review, descriptive research, interviews, and difficulty analysis to study solid geometry questions. The solid geometry questions in the college entrance examination are stable, but relying solely on practice to improve scores does not adequately reflect the true purpose of education in our country. The study found that the types of solid geometry questions tested in the college entrance examination include structural characteristics and three views, surface area and volume of spatial geometric solids, circumscribed and inscribed spheres, line-plane relationships, and spatial angles. This paper analyzes and summarizes problem-solving techniques and strategies to cultivate students problem-solving thinking and help them learn more effectively, avoiding the rigid practice of solving countless problems.*

Keywords: Solid geometry, Difficulty analysis, Test research.

1. Research Background

The college entrance examination, also known as the National Unified Examination for Admission to Ordinary Higher Education Institutions, serves as a summary of students three years of high school and is one of the fair and just ways to select talent for the country. Mathematics has always held an extremely important position throughout history, which is one of the major reasons why it is a required subject in the college entrance examination.

In mathematics, solid geometry is one of the traditional names for three-dimensional Euclidean space geometry, which essentially represents our living space. Solid geometry courses follow the study of plane geometry. Additionally, as one of the key points in college entrance examination mathematics, solid geometry typically accounts for an average of 22.8 points in each set of real exam questions, making up 15.2% of the total score. The National Paper I/II/III usually includes 2 minor questions and 1 major question, so solid geometry is essential content for students to master.

2. Research Methods

2.1 Literature Review Method

The literature method is an ancient and widely applied scientific research approach. Various documents are important records and reflections of social activities. Therefore, this paper summarizes the compilation of solid geometry textbooks, the examination and teaching methods of solid geometry under core competencies, the key points of solid geometry in college entrance examination questions over the past five years, as well as problem-solving ideas and

methods through reading, analyzing, and organizing relevant literature materials.

2.2 Descriptive Research Method

Describe and summarize the college entrance examination questions involved in the selected topics through your own understanding and verification.

2.3 Comprehensive Difficulty Model

The comprehensive difficulty model of Wu Xiaopeng was used to calculate the difficulty coefficient of nine factors that affect the test questions, including background factor, parameter level, operation level, reasoning ability, knowledge content, problem solving thinking mode, cognitive level, condition content and character reading volume.

3. Research Results

3.1 Statistics of the Form and Value of Solid Geometry Questions in the College Entrance Examination in Recent Five Years

This paper primarily focuses on the solid geometry questions in the National Mathematics I, II, and III Papers (Science) from 2018 to 2022, analyzing the characteristics of these questions and summarizing problem-solving methods and approaches. The paper also examines the structure of the test, noting that the total score for the college entrance examination mathematics is 150 points, with question types divided into multiple-choice, fill-in-the-blank, and subjective questions. Multiple-choice and fill-in-the-blank questions are worth 5 points each, totaling 16 questions, while subjective questions

are worth 12 points each, totaling 6 questions. Solid geometry questions cover multiple-choice, fill-in-the-blank, and subjective questions, with the proportion of question types varying each year. According to the statistical chart of solid

geometry-related questions over the past five years, it can be observed that the question types tested in the college entrance examination are relatively stable, especially for multiple-choice and subjective questions.

Table 1: Statistics of solid geometry questions in college entrance examination papers in recent five years

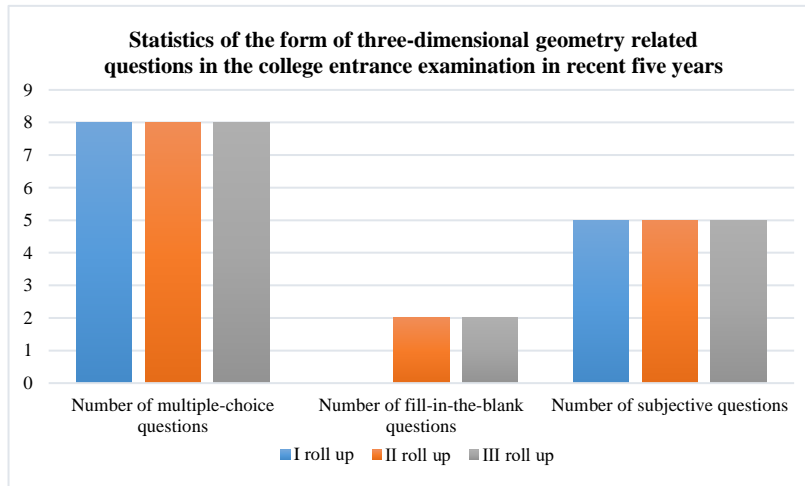| examination paper	Number of multiple-choice questions	Number of fill-in-the-blank questions	Number of subjective questions	total quantity	Proportion of the number of questions	Percentage of points
I roll up	8	0	5	13	11.30%	11.33%
II roll up	8	2	5	15	13.04%	14.67%
III roll up	8	2	5	15	13.04%	14.67%
total quantity	24	4	15	43		

Figure 1: Statistics of three-dimensional geometry related questions in the college entrance examination in recent five years
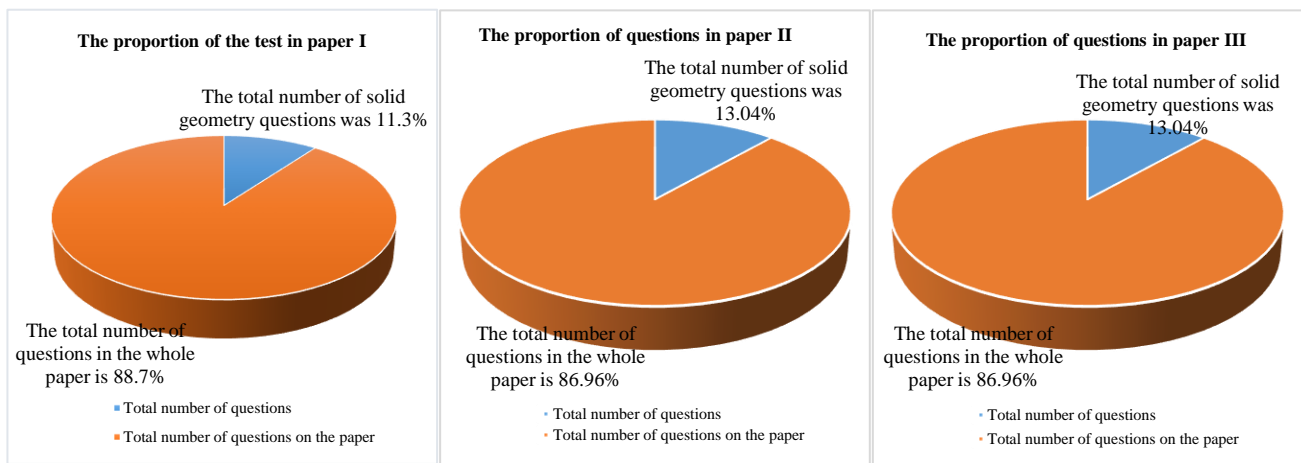

Figure 2: The proportion of solid geometry in the college entrance examination I, II and III in the past five years

According to Figure 2, the three-dimensional geometry knowledge points examined in the national test are 11.3% in paper I, 13.04% in paper II and 13.04% in paper III. The number of three-dimensional geometry questions is relatively stable, and the number of questions examined is also relatively stable.

The quantity is small. Therefore, students can quickly master this part of the content, and can also study the solid geometry questions.

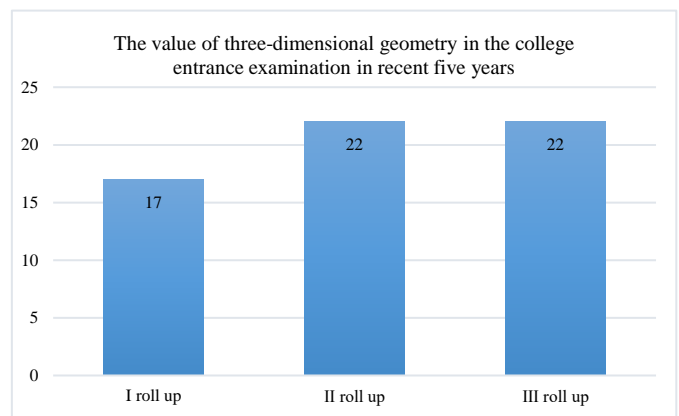To find the corresponding test point problem solving skills, so as to help students improve their grades.

Figure 3: Statistics of the score of solid geometry in the college entrance examination in recent five years

According to the statistical data on solid geometry scores in the national I, II, and III college entrance examination papers over the past five years, it is clear that multiple-choice questions are worth 5 or 10 points, while fill-in-the-blank questions are worth 0 or 5 points. Additionally, subjective questions always carry 12 points each year. Therefore, solid geometry problems are a fixed and essential part of the college entrance examination. Moreover, the score for these questions remains relatively stable at 22 points annually, with a few cases scoring 17 points.

Combining the statistics of solid geometry scores from college entrance examination papers over the past five years, it can be seen that solid geometry holds a significant position in the mathematics section of the college entrance exam, with relatively high score allocations. High school students need to understand the importance of this topic and master solid geometry proficiently. At the same time, teachers should place greater emphasis on the teaching design of this content, so that students can achieve high scores in solid geometry problems in the college entrance exam mathematics.

3.2 Statistics of the Types of Solid Geometry Questions and the Frequency of Each Test Point in the College Entrance Examination Papers in the Past Five Years

The key points involved in the three-dimensional geometry section of the college entrance examination mathematics include: characteristics of simple three-dimensional geometric structures, three views of simple geometric solids, relationships between spatial points, lines, and planes, operations with spatial vectors, and solving three-dimensional geometry problems using spatial vectors. The table below analyzes the types of three-dimensional geometry questions examined in the national I, II, and III papers over the past five years.

Table 2: Distribution of solid geometry questions in the national I paper in the past five years

a particular year	choice question	gap filling	subjective item	total value
2018	three views	\	Face perpendicular, line and surface angle	17
2019	circumscribed sphere	\	Parallel lines and planes, dihedral angles	17
2020	Surface area of a quadrilateral pyramid and sphere	\	Line and plane perpendicular, dihedral angle	17
New college entrance exam in 2021	Conical generatrix, spatial vector	\	The lines are perpendicular and the volume of the triangular pyramid	22
New college entrance examination in 2022	Volume of geometric body, line and surface relationship	\	Distance from a point to a plane, dihedral Angle	22

Table 3: Distribution of solid geometry questions in the national paper II in the past five years

a particular year	choice question	gap filling	subjective item	total value
2018	Spatial vectors	Surface area of a cone	Line and plane perpendicular, line and plane angle	22
2019	Parallel on all sides	Simple geometry	Line and plane perpendicular,	22

Table 4: Distribution of solid geometry problems in national paper III in the past five years

a particular year	choice question	gap filling	subjective item	total value
2020	Three views, sphere	\	dihedral angle Parallel lines, line plane angles	22
New college entrance examination in 2021	Volume of geometric body, perpendicular line and surface	\	Face perpendicular, dihedral angle	22
New college entrance examination in 2022	Volume of a sphere, geometric body	\	Parallel lines and planes, dihedral angles	22
2018	Three views, geometric volume	\	Face perpendicular, dihedral angle	22
2019	The positional relationship between lines	Volume of a solid	Face perpendicular, dihedral angle	22
2020	three views	Volume of geometric bodies	The position of points and planes, the dihedral angle	22
2021 Paper A	Three views, geometric volume	\	The lines are vertical and the minimum sine value	22
2022 Paper A	Three views, geometric volume	\	Line perpendicular, line plane angle	22

Solid geometry is a crucial concept in high school mathematics and a frequently tested topic in the college entrance examination. To facilitate the analysis of the weight distribution of various knowledge points in the exam, this article categorizes the key points of solid geometry into four major question types: structural characteristics and three views of solid geometry, surface area and volume of spatial geometric solids and related calculations, circumscribed and inscribed spheres, and line-plane relationships and spatial angles. By analyzing these four major question types and statistically examining the number of questions, we can clearly identify the focus areas for solid geometry assessment. The specific statistics are as follows:
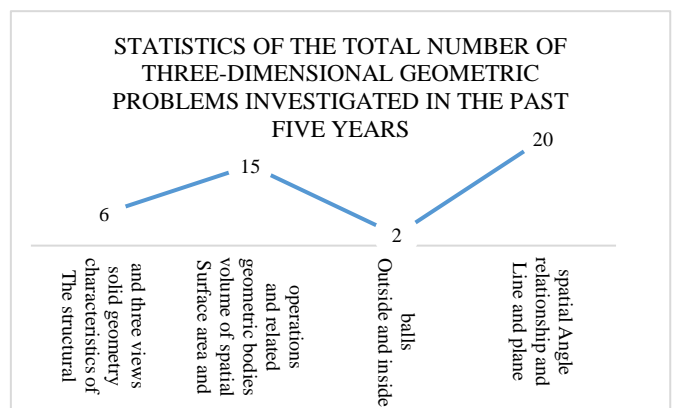

STATISTICS OF THE TOTAL NUMBER OF THREE-DIMENSIONAL GEOMETRIC PROBLEMS INVESTIGATED IN THE PAST FIVE YEARS

Figure 4: The total number of three-dimensional geometry questions in the college entrance examination in recent five years
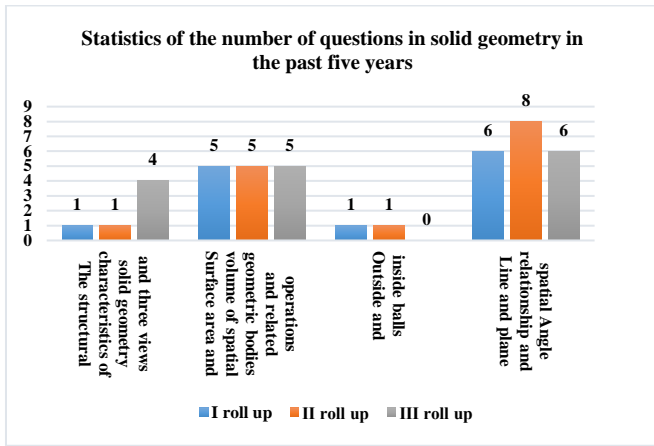
Figure 5: Statistics of the number of questions in solid geometry in the past five years

Based on the above statistical analysis, in the past five years, multiple-choice questions in the national I, II, and III exams have mainly focused on three-dimensional views; surface area and volume of simple geometric solids; and positional relationships between points, lines, and planes in space. In the past five years, fill-in-the-blank questions have relatively stabilized at examining the structural characteristics of simple geometric solids. Subjective questions are divided into two sub-questions each year, with clear test points and stable question types. The first sub-question primarily examines the positional relationships between points, lines, and planes in space, where proving the positional relationship between a line and a plane is an important and common test point, accounting for as high as 87% of the 15 sets of questions. The second sub-question mainly examines the sine values of angles between lines and planes, as well as the sine values of dihedral angles. In summary, the surface area and volume of spatial geometric solids and related calculations, as well as line-plane relationships and spatial angles, are common test types in the college entrance examination over the past five years.

Table 5: Analysis of the difficulty of three-dimensional geometry questions in the national I paper in the past five years

a particular year	The difficulty of the multiple choice questions	Fill in the blanks	Subjective questions are difficult	total value
2018	Basic questions	\	Medium difficulty	17
2019	difficult problem	\	Medium difficulty	17
2020	Basic questions	\	Medium difficulty	17
New college entrance examination in 2021	Basic questions, difficult questions	\	difficult problem	22
New college entrance examination in 2022	Medium and basic questions	\	Medium difficulty	22

Table 6: Analysis of the difficulty of three-dimensional geometry questions in the national paper II in the past five years

a particular year	The difficulty of the multiple-choice questions	Fill in the blanks	Subjective difficulty	total value
2018	Basic questions	difficult problem	Medium difficulty	22
2019	Basic questions	Medium difficulty	Medium difficulty	22
2020	Basic questions, medium questions	\	difficult problem	22
New college entrance examination in 2021	Basic questions, medium questions	\	Medium difficulty	22
New college entrance examination in 2022	Basic questions, medium questions	\	Medium difficulty	22

Table 7: Analysis of the difficulty of three-dimensional geometry questions in the national iii paper in the past five years

| a particular year	The difficulty of the multiple choice questions	Fill in the blanks	Subjective questions are difficult	total value
2018	Basic questions, medium questions	\	Medium difficulty	22
2019	Basic questions	difficult problem	Medium difficulty	22
2020	Basic questions	Medium difficulty	Medium difficulty	22
New college entrance examination in 2021	Basic questions, medium questions	\	Medium difficulty	22
New college entrance examination 2022	Basic questions, basic questions	\	Medium difficulty	22
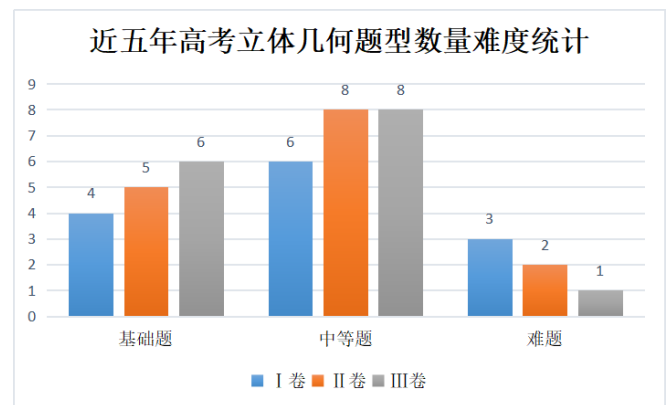
Figure 6: Statistics of the number and difficulty of solid geometry questions in the college entrance examination in recent five years
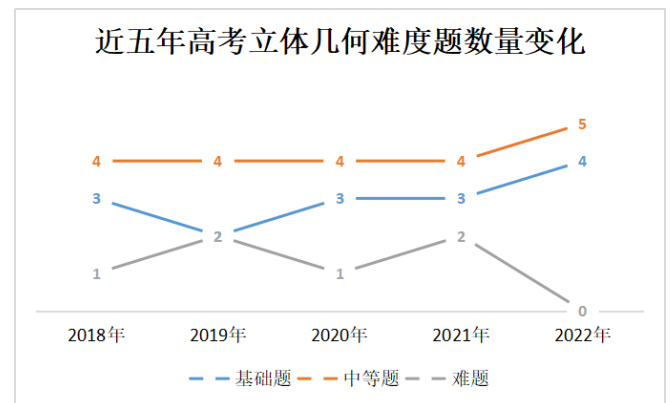

Figure 7: The line graph shows the change of the number of difficult problems in solid geometry in the college entrance examination in recent five years

Combined with the above analysis of the difficulty of the questions, there are some basic questions in the

multiple-choice questions every year, and the fill-in-the-blank questions are mostly medium or difficult.

For subjective questions, the focus is mainly on medium-level ones, so the difficulty level is stable and consists primarily of basic and medium-level questions each year. As shown in Figures 7, the number of basic and medium-level questions remains relatively stable and accounts for a high proportion. On the other hand, difficult questions are less frequently tested each year, and no challenging questions appeared in any of the three sets of papers in 2022. Therefore, it can be concluded that the difficulty of solid geometry in the college entrance examination has been decreasing year by year. This is good news for high school students, as long as they grasp the key concepts, they can score relatively easily.

In summary, the types and difficulty of solid geometry questions in the college entrance examination are relatively stable. Therefore, this paper will analyze these types of questions and summarize problem-solving ideas and methods, further assisting teachers in providing more targeted explanations of key points during the senior year review stage. It will also help students better understand the direction of solid geometry questions and confidently score this section in the college entrance examination.

3.3 Analysis of the Three-dimensional Comprehensive Difficulty of the College Entrance Examination Papers in the Past Five Years

It is important to clearly analyze the composition and comprehensive difficulty of test questions. A good set of questions should have an appropriate proportion of difficulty. Therefore, to better grasp the importance of knowledge points, it is necessary to analyze both the form and difficulty of test questions.

3.3.1 "Solid geometry" comprehensive difficulty model

The comprehensive difficulty model is a method that uses hierarchical weights to measure and statistically analyze the difficulty of test questions, making it one of the key tools for analyzing test difficulty. Therefore, this method plays a significant role in comparative education research. The concept of overall difficulty was first introduced by Nohara in an educational statistics report. Subsequently, Bao Jiansheng and others adjusted the model to derive five difficulty factors: exploration, background, computation, reasoning, and knowledge content. However, Bao Jianshengs comprehensive difficulty model did not delve deeply enough into these five factors. In this paper, Wu Xiaopengs improved model combines the characteristics of middle school and high school entrance exams with Bao Jianshengs five difficulty factors, resulting in a model that includes: background factor, parameter level, computational level, reasoning ability, knowledge content, problem-solving mindset, cognitive level, condition content, and character reading volume.

3.3.2 Comprehensive difficulty analysis of "Solid Geometry"

Using the background factors, parameter levels, computational levels, reasoning abilities, knowledge content, problem-solving thinking patterns, cognitive levels, condition content, and character reading volume categorized by the comprehensive difficulty model to analyze the difficulty of national exams from 2018 to 2022. From the analyzed difficulties, we can identify the key points and directions of examination in three-dimensional geometry questions for the college entrance exam, which allows for more effective and accurate analysis of test points and difficulty levels, thereby facilitating the rapid summarization of problem-solving techniques and strategies. In this statistical analysis, each set of exam papers containing two sub-questions on three-dimensional geometry is combined into one question for statistical analysis. The following is the calculation method using Wu Xiaopengs comprehensive difficulty model[21]The results of the statistics are as follows:

Table 8: Comprehensive statistics of national college entrance examination mathematics (science) test papers in the past five years

factor	horizontal	Number of questions			percentage			Combine the difficulty coefficient		
		I roll up	II roll up	III roll up	I roll up	II roll up	III roll up	I roll up	II roll up	III roll up
background factor	Background free	12	15	14	92.30%	100%	93.33%	0.22	0.20	0.22
	Life background	1	0	1	7.70%	0.00%	6.67%			
	Scientific background	0	0	0	0.00%	0.00%	0.00%			
	Background	0	0	0	0.00%	0.00%	0.00%			
parameter horizontal	No parameters	6	8	6	46.15%	53.33%	40.00%	0.43	0.45	0.49
	There are simple parameters	5	3	4	38.46%	20.00%	26.67%			
	There are complex parameters	2	4	5	15.39%	26.67%	33.33%			
operation horizontal	Simple numbers	6	5	3	46.15%	33.33%	20.00%	1.35	1.37	1.40
	Complex numbers	6	7	11	46.15%	46.67%	73.33%			
	Simple symbols	3	4	3	23.08%	26.67%	20.00%			
	Complex symbols	5	5	5	38.46%	33.33%	33.33%			
inference ability	Simple reasoning	3	5	3	23.08%	33.33%	20.00%	1.09	0.90	0.97
	General reasoning	4	5	7	30.77%	33.33%	46.67%			
	Complex reasoning	6	5	5	46.15%	33.34%	33.33%			
knowledge content	A small amount of knowledge	6	8	8	46.15%	53.33%	53.33%	0.78	0.73	0.68
	Medium knowledge points	4	6	7	30.77%	40.00%	46.67%			
	A lot of knowledge	3	1	0	23.08%	6.67%	0.00%			

	points									
Problem solving thinking	Think in the right direction	11	14	14	84.62%	93.33%	93.33%	0.58	0.54	0.54
	reversed thinking	2	1	1	15.38%	6.67%	6.67%			
knowledge horizontal	memorize	4	8	8	30.77%	53.33%	53.33%	1.06	1.03	1.06
	utilize	4	3	2	30.77%	20.00%	13.33%			
	aggregate analysis	5	4	5	38.46%	26.67%	33.34%			
condition content	Single condition	3	2	1	23.08%	13.33%	6.66%	1.22	1.24	1.26
	Two conditions	5	6	7	38.46%	40.00%	46.67%			
	Three or more	5	7	7	38.46%	46.67%	46.67%			
character read measure	A small number of characters	9	8	9	69.23%	53.33%	60.00%	0.79	0.83	0.81
	Medium character	4	6	5	30.77%	40.00%	33.33%			
	A lot of characters	0	1	1	0.00%	6.67%	6.67%			

3.3.2.1 Comparison of different levels of factors
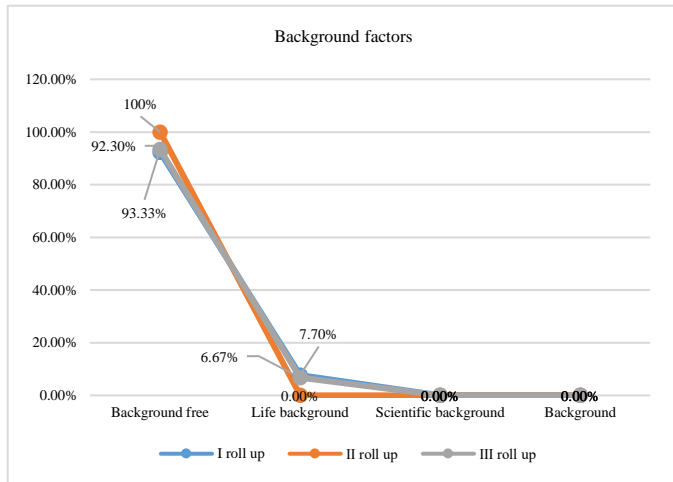
Figure 8: Background factor level change line graph

1) In the background factor line graph, it is clear that in the past five years, the three-dimensional geometry questions in the college entrance examination have almost never involved scientific or real-life contexts. Only one question in the third paper involved a real-life context, indicating that this aspect is relatively underrepresented. The proposition of questions should still integrate real-life and scientific contexts with the problems to help students understand that mathematics is closely related to life and that mathematics and science contribute to national progress. This will help students realize the importance of learning mathematics.
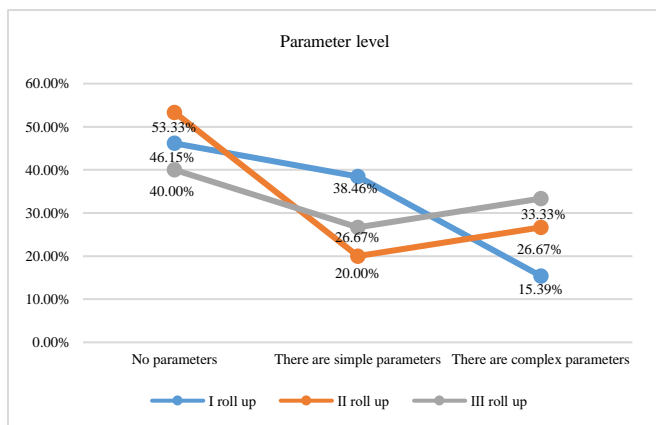

Figure 9: Parameter level change line graph

2) In the parameter difficulty line graph, we can get that both simple and complex parameters are involved in the national exam, but

More questions have no parameter setting, and the difficulty

distribution is more uniform. Therefore, students can easily master such questions without a large number of parameter setting and solution.
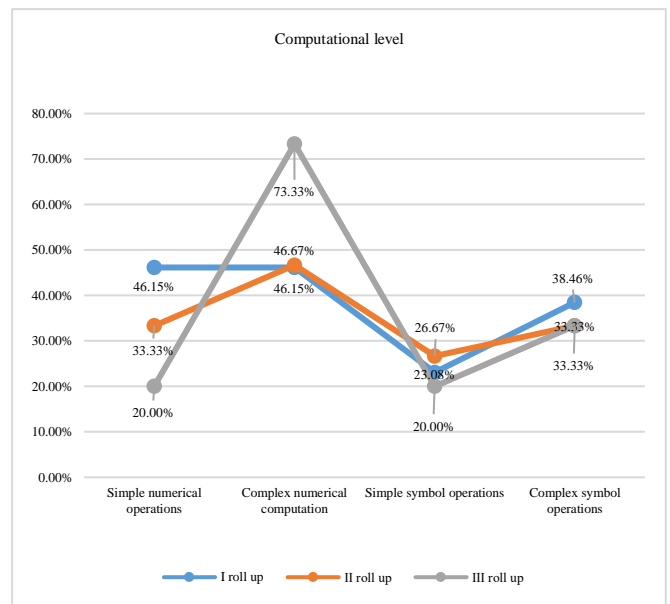

Figure 10: Line graph of changes in operational level

3) The difficulty of calculations can be clearly seen from the line graph, which shows that most questions in the national exam focus on simple numerical and symbolic operations. Only a few questions involve complex numerical and symbolic calculations. Therefore, students should not take solid geometry problems lightly and must also focus on improving their calculation skills to avoid errors in complex problem-solving. At the same time, they should ensure that they do not make mistakes in simple numerical and symbolic operation questions, thus easily securing the required scores.
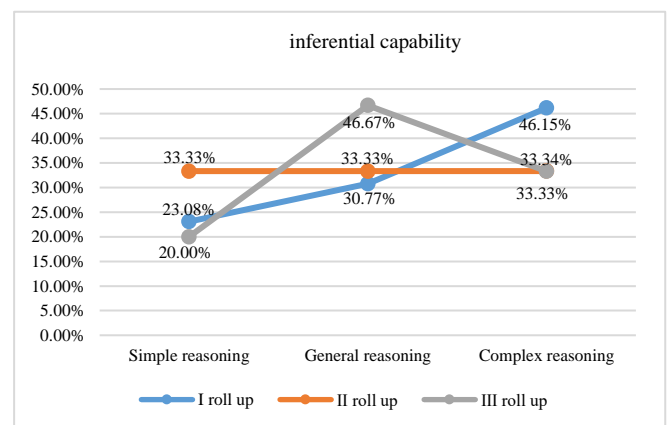

Figure 11: Line graph of changes in reasoning ability

4) The line graph of reasoning difficulty reveals that solid geometry problems often involve a large number of steps in solving them. Students tend to make mistakes in these steps, so it is important to cultivate clear reasoning skills. This is especially true for the subjective questions in solid geometry, which require a clear problem-solving approach and the complete, correct, and organized writing of each step. The specific steps and techniques for solving subjective questions will be covered in Chapter Three, helping students write their solutions accurately.
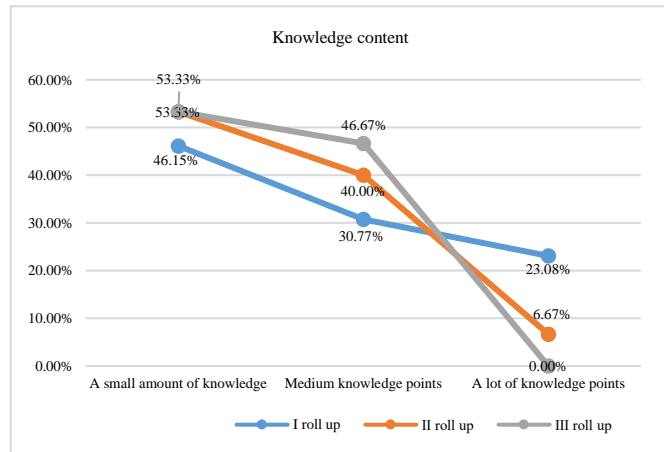
Figure 12: 0Knowledge content change line graph

5) The line graph of knowledge points shows that the national exam questions on solid geometry cover fewer knowledge points. It is unlikely for a single question to involve a large number of knowledge points, which could confuse students. Moreover, most questions test relatively simple knowledge points. Students only need to identify the key points based on the given conditions in the questions to solve them quickly. At the same time, students should master the knowledge points of solid geometry in their regular studies.
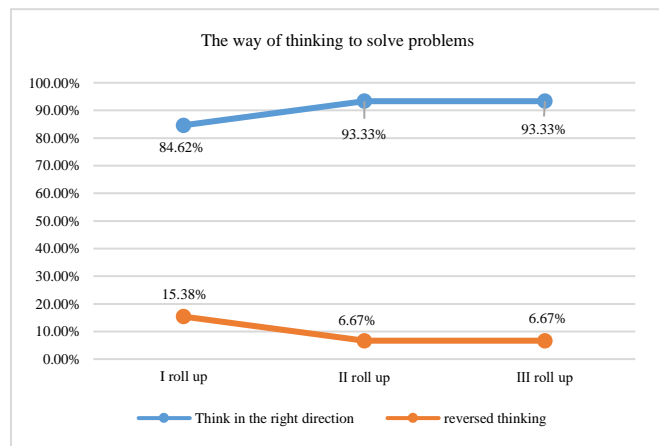

Figure 13: 1 The way of thinking to solve the problem is shown in a line graph

6) In the line graph of thinking patterns, it is clear that the questions primarily test forward thinking, with very few testing reverse thinking. Therefore, solid geometry problems require students to consider issues from the perspective of the given problem, seeking solutions to the problem using a forward-thinking approach. The key feature is the continuous exploration of solutions while maintaining the established line of thought.
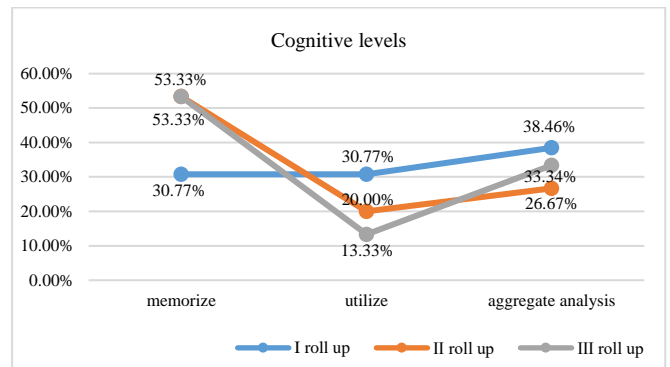

Figure 14: Line graph of cognitive level change

7) In the cognitive difficulty line graph, it is found that the comprehensive level of questions in the national exam is relatively stable. It indirectly examines knowledge and requires flexible application. However, most questions directly test one knowledge point or indirectly test several knowledge points, meaning only a few knowledge points need to be comprehensively applied. These questions mainly assess students ability to analyze and solve problems comprehensively.
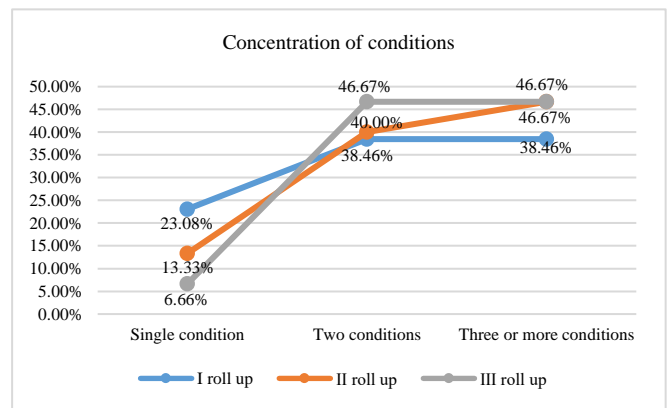

Figure 15: Line graph of condition content change

8) The line graph of conditions shows that a large number of questions contain 2 or more conditions, while only a few questions have just one condition. Questions with a single condition are generally simpler, allowing students to quickly grasp the key points and solve the problem. For questions with multiple conditions, students need to think carefully, making good use of each condition to ensure accuracy in their answers.
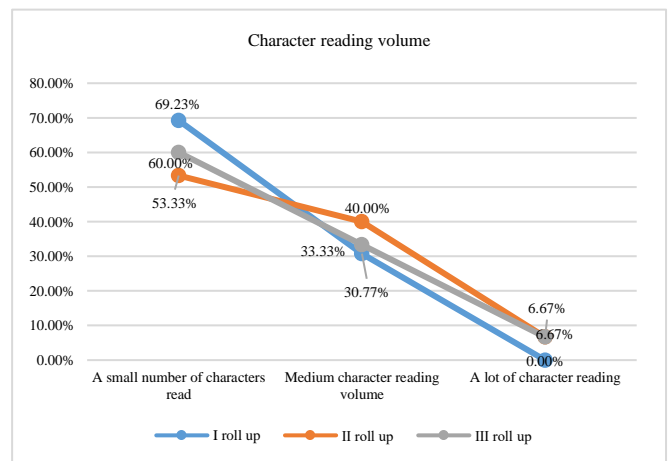

Figure 16: Line graph comparing the reading volume of each character

9) The line graph of character count shows that the number of characters in solid geometry questions on the national exam is relatively low, making it easier for students to read quickly and focus on key points for solving problems. For questions with more characters, students should develop the habit of drawing diagrams while reading the question, so they can quickly identify the required conditions from the stem after finishing the reading.
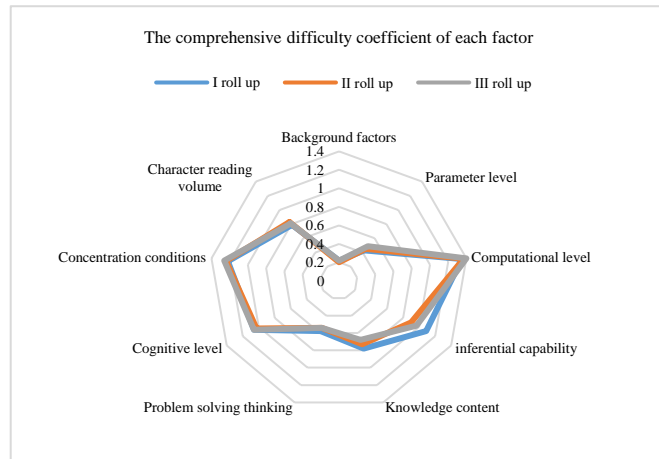
3.3.2.2 Comprehensive difficulty comparison

Figure 17: Radar chart of comprehensive difficulty coefficient of "Solid geometry" questions in the national college entrance examination mathematics test in the past five years

The radar chart analysis is as follows:

Among the nine factors affecting the difficulty of solid geometry problems, it is evident that background factors, parameter levels, and problem-solving thinking have the least impact; the number of characters and the difficulty coefficient of knowledge points are at a moderate level; while the number of conditions, cognitive difficulty, reasoning ability, and computational difficulty coefficients are more significant. Therefore, these four influencing factors have the greatest impact on the difficulty of solid geometry problems.

The background factors, parameter levels, condition content, cognitive levels, problem-solving thinking patterns, computational skills, and character reading difficulty coefficients of solid geometry questions in the national I, II, and III papers are not significantly different, with their difficulty levels being roughly equal. In terms of knowledge content difficulty coefficients, Paper I is more difficult than Paper III, and Paper III is more difficult than Paper II; regarding reasoning ability difficulty coefficients, National Paper I is more difficult than National Paper II, and National Paper II is more difficult than National Paper III.

Project Fund:

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