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Study on Solving Problems of Conic Sections in the New College Entrance Examination — Take the National Test in 2019-2024 as an Example

Gang Xiao[1], Hongying Xiao[2,*], Hongmei He[3]

[1]Master/Lecturer, School of Mathematics and Physics, Yibin University
[2]School of Mathematics and Physics, Yibin University, PhD/Associate Professor
[3]Yibin No.3 Middle School, master/ middle school student
Correspondence Author

Abstract: ***With the gradual implementation of the new college entrance examination, Sichuan, as the fifth echelon of the reform, will also officially open the first new college entrance examination this year. However, the change of question type and the increase of difficulty of test questions also follow. The conic curve test is one of the key and difficult points in high school mathematics, and it is also the pride of the new college entrance examination. The knowledge of conic curve is an important part of plane analytic geometry, and it is also a typical combination of number and shape. It is self-evident that it contains mathematical ideas. Based on the existing relevant research, this paper studies the conic curve test questions in the national volume of the new college entrance examination from 2019 to 2024. The content of this paper can be divided into three parts: The first part introduces the research background, research questions, and expounds the purpose and significance of the research. This paper introduces the related research of many scholars, including the content of conic curve teaching materials, problem-solving errors, problem-solving strategies, teaching and so on. It also expounds Polya s problem-solving theory and the conic curve content of the new college entrance examination, based on which the analysis is carried out. The second part, as the core part of this paper, studies the six-year conic curve test questions from two perspectives. Firstly, it is the requirement of mastering the conic curve in the new curriculum standard. Secondly, it is the analysis of the proportion of test questions and scores, the test points and difficulty of the test questions. Finally, it is a comprehensive analysis. The collected real questions are sorted out, classified and summarized into the corresponding questions, and the corresponding problem-solving analysis and strategies are given. The third part summarizes the work of this paper, and puts forward some suggestions for teachers teaching and students learning.***

Keywords: New college entrance examination, Conic curve, Question type analysis, Problem solving strategy.

1. Research Background

Conic sections are a crucial component of high school analytic geometry and a core focus and challenge in the college entrance examination. In terms of score distribution, this section carries significant weight in the mathematics exam, typically ranging from 17 to 22 points in recent years, and reaching as high as 28 points in the 2024 national paper. In terms of coverage, the knowledge involved is extensive, appearing in multiple-choice questions, multiple-selection questions, fill-in-the-blank questions, and problem-solving questions. On the national college entrance examination math paper, conic section problems usually appear as the second-to-last or final major question. From an examination perspective, the difficulty levels are clearly differentiated. Multiple-choice questions, multiple-selection questions, and fill-in-the-blank questions primarily test students understanding and mastery of basic knowledge, such as the definitions of ellipses, parabolas, and hyperbolas, their grasp of simple geometric properties, and their ability to apply formulas. Problem-solving questions have significantly increased in difficulty, becoming more comprehensive and abstract, emphasizing students overall abilities and often integrating content from other chapters.

2. Research Problems and Methods

2.1 Research Questions

The research object of this paper is the new college entrance examination, which is the national high school conic section test from 201 9 to 2024. The main research problem is:

(1) Through the analysis of test questions, this paper analyzes the national high school conic section questions from 201 9 to 2024 in terms of question type, test point and value proportion.

(2) Summarize and categorize the collected high school conic section problems from the national exams for 2 01 9-2024, identifying key test points: trajectory and trajectory equation problem-solving, geometric properties of conic sections, positional relationship between lines and conic sections, intersecting chord problems of conic sections, fixed point problems of conic sections, value determination problems of conic sections, maximum and minimum value and range problems of conic sections, and comprehensive problems involving conic sections. For each major test point, select typical problems to analyze solution strategies and summarize problem-solving methods.

2.2 Research Methods

Research methods are systematic processes and techniques used in research to collect, analyze, and interpret data or information, serving as means to explore the inherent laws of things. Common methods include literature review, case study, survey research, experimental research, comparative research, interview, interdisciplinary research, and action research. The primary research methods adopted in this paper are literature review and theoretical research.

3. Research Results

From four aspects—analyzing the problem, analyzing the solution method, evaluating and reflecting (analyzing the problem from three aspects: background, key points, and difficulty level); for the several major common examination points that have been organized and categorized: trajectory and trajectory equations, geometric properties of conic sections, positional relationships between lines and conic sections, intersecting chords, fixed points, fixed values, maximum and minimum values and ranges, comprehensive analysis of conic sections, to summarize and outline corresponding problem-solving ideas and methods.

3.1 Solution of Trajectory and Trajectory Equation Problem

Example 1: (202 4 National Paper II, Question 5)

Known curve C: (), draw a perpendicular line from any point P on C to the x-axis, and the midpoint M of the line segment is ()$x^2+y^2=16y>0$ PP' $P'PP'$

A. () $\frac{x^2}{16}+\frac{y^2}{4}=1y>0$
B. () $\frac{x^2}{16}+\frac{y^2}{8}=1y>0$
C. $\frac{y^2}{16}+\frac{x^2}{4}=1$ () $y>0$
D. () $\frac{y^2}{16}+\frac{x^2}{8}=1y>0$

3.1.1 Analyze the questions

This problem is based on the concept of ellipses and trajectory equations, with a simple difficulty level. It primarily tests the method of substitution to find the trajectory equation. When there is a certain relationship between the coordinates of the moving point being sought and those of the moving point on a known curve, one can set up the coordinates of the moving point being sought, use the given conditions to find its relationship with the coordinates of the moving point on the known curve, then substitute the coordinates of the moving point on the known curve into the equation of the known curve, i.e., set up the point, and according to the problems requirements, express the coordinates of the midpoint, thus obtaining the trajectory equation of the moving point being sought. $M(x,y)$ $P(x,2y)$

3.1.2 Solution analysis (standard answer)

1) Let the desired point be set as, then; $M(x,y)P(x,y_0),P'(x,0)$

2) According to the meaning of the problem, it is known that the midpoint is, then, that is, M PP' $y_0=2y$ $P(x,2y)$

3) And because it is on a circle, so; P $x^2+y^2=16(y>0)$ $x^2+4y^2=16(y>0)$ $\frac{x^2}{16}+\frac{y^2}{4}=1(y>0)$

4) Then the trajectory equation of the point is. M $\frac{x^2}{16}+\frac{y^2}{4}=1(y>0)$

Therefore, the answer is A

3.1.3 Evaluation and reflection

Common methods for finding the equation of a trajectory include direct method, definition method, related points method, and parameter method. This question focuses on the core knowledge point of using the related points method to find the equation of a trajectory, requiring candidates to have a deep understanding of the concepts of curves and equations; at the same time, it also tests logical reasoning and mathematical abstraction skills, with a reasonable difficulty level set for basic questions. However, the form of the question is relatively traditional, and the methods used to find the equation of a trajectory are quite conventional, lacking some innovation.

3.2 Geometric Properties of Conic Sections

Example 2: (202 4 National Paper II, Question 10)

Parabola C: the asympote of C is l, P is a moving point on C, a tangent line through P, Q is the tangent point, and a perpendicular line through P to l, the foot of which is B, then ()$y^2=4x\odot A{:}x^2+(y-4)^2=1$

A. l is tangent to the circle $\odot A$
B. When P, A, B are collinear, $|PQ|=\sqrt{15}$
C. then, $|PB|=2PA\perp AB$
D. There are exactly two points of satisfaction $|PA|=|PB|$ P

3.2.1 Analyze the questions

This problem is based on the intersection of a line and a parabola, as well as the length of tangents. It involves solving for the focus or directrix using the equation of the parabola, with a moderate level of difficulty. The main focus is on testing students understanding of the geometric properties of parabolas, the positional relationship between lines and parabolas, distance and perpendicularity concepts, and their ability to reason about distances in geometric figures. In the new college entrance examination, there are multiple-choice questions ——. For this question, we should use the geometric properties of the parabola to determine that the directrix is, then judge the positional relationship between them based on the distance from the center of the circle to the directrix; for determining the length of the tangent, we can first find the coordinates when three points lie on the same line, and then derive the result accordingly; we can also verify whether it holds true based on the coordinates calculated first; finally, according to the definition of a parabola, the problem is transformed into a question about the existence of points, at which point the number of intersections between the perpendicular bisector and the parabola can be examined, or we can directly set the coordinates of the point to solve the problem.$x=-1P,A,B$ P $|PB|=2$ P $k_{PA}k_{AB}=-1$ $|PB|=|PF|$ $|PA|=|PF|P$ AF P

3.2.2 Solution analysis (standard answer)

1) Option A is true

(a) It is known that the parabola is then the direct line is; $y^2=$

$4xx = -1$

(b) It is known that the distance from the center to the line is obviously equal to the radius of the circle; $\odot A: x^2 + (y-4)^2 = 1(0,4)x = -11$

(c) Therefore, the normal line and tangent are correct, option A is correct; $l \odot A$

2) Option B is true

(a) If three points are collinear, then the vertical coordinate of is; $P, A, BPA \perp l\ P\ y_P = 4$

(b) Again, to get, therefore;$y_P^2 = 4x_P\ x_P = 4\ P(4,4)$

(c) At this time, the tangent length is correct, option B;$|PQ| = \sqrt{|PA|^2 - r^2} = \sqrt{4^2 - 1^2} = \sqrt{15}$

3) Option C is true

(a) If known, then, so or; $|PB| = 2\ \ x_P = 1\ \ y_P^2 = 4x_P = 4P(1,2)\ P(1,\text{-}2)$

(b) Further classification discussion:

At that time,,,, was not satisfied;$P(1,2)\ A(0,4), B(\text{-}1,2)\ k_{PA} = \frac{4-2}{0-1} = -2\ k_{AB} = \frac{4-2}{0-(-1)} = 2\ k_{PA}k_{AB} = -1$

then,,,, $P(1,2)\quad A(0,4), B(-1,2)\quad k_{PA} = \frac{4-2}{0-1} = -6\quad k_{AB} = \frac{4-(-2)}{0-(-1)} = 6$

dissatisfaction; $k_{PA}k_{AB} = -1$

(c) Therefore, it is not established, option C is wrong; $PA \perp AB$

4) Option D is true

Method 1: Transformation of parabolic definition

(a) According to the definition of a parabola, and known, $|PB| = |PF|F(1,0)$

(b) Then the transformation of the problem proved by the title: the existence of the time point is transformed into the existence of the time point; and, the slope of the midpoint and the perpendicular line is; $|PA| = |PB|\ P\ |PA| = |PF|PA(0,4), F(1,0)\ AF\left(\frac{1}{2}, 2\right)\ AF - \frac{1}{k_{AF}} = \frac{1}{4}$

(c) Then the equation of the central perpendicular line is:, and the parabola can be combined to get. According to, we know that the central perpendicular line and the parabola have two intersection points; $AF\ y = \frac{2x+15}{8}\ y^2 = 4x\ y^2 - 16y + 30 = 0\Delta = 16^2 - 4 \times 30 = 136 > 0\ AF$

(d) That is, there are two points such that option D is correct. $P\ |PA| = |PF|$

Method 2: Direct point solving method

(a) Let, so that, and, and; $P\left(\frac{t^2}{4}, t\right) PB \perp lB(-1, t)\ A(0,4)\ |PA| = |PB|$

(b) Then by the distance formula between two points, we can get, $\sqrt{\frac{t^4}{16} + (t-4)^2} = \frac{t^2}{4} + 1\ t^2 - 16t + 30 = 0$

$\Delta = 16^2 - 4 \times 30 = 136 > 0$ Then the equation has two solutions; t

(c) There are two such points, option D is correct. P

Therefore, A BD

3.2.3 Evaluation and reflection

This question is comprehensive, skillfully integrating several key concepts such as the geometric properties of parabolas and distance relationships within figures. It not only tests basic knowledge like the equation and definition of a parabola but also explores the positional relationship between a line and a parabola, covering a wide and systematic range of knowledge. The examination of thinking skills is more profound compared to basic questions, presenting a certain level of difficulty. In terms of logical reasoning, it requires students to start from known conditions, gradually deduce the connections between points and distances, and reach conclusions through rigorous logical reasoning; in combining geometric intuition with algebraic operations, students need to construct the positional relationships of geometric figures such as parabolas, tangents, and normals in their minds, while using algebraic methods for differentiation, equation solving, and other calculations, organically integrating geometric intuition with algebraic operations to enhance mathematical thinking quality and cultivate the ability to think and solve problems from different perspectives. The overall difficulty of the question is moderate, reflecting a comprehensive assessment of core high school mathematics knowledge, which helps evaluate students overall mastery and integrated application skills. Compared to traditional college entrance exams, the question format has been somewhat improved, changing from single-choice to multiple-choice, thus increasing the amount of knowledge required.

3.3 Position Relationship between Straight Line and Conic Section

Example 3: (202 3 National B paper (science) No.11)

Let A and B be two points on the hyperbola, among the following four points, () can be the midpoint of the line segment AB $x^2 - \frac{y^2}{9} = 1$

A. $(1,1)$
B. $(-1,2)$
C. $(1,3)$
D. $(-1,-4)$

3.3.1 Analyze the questions

This problem is set against the background of determining the position of a line relative to a conic section using the midpoint chord of a hyperbola. The key to solving it lies in utilizing the properties of hyperbolas and the point difference method to determine whether the given point can be the midpoint of a line segment. By solving systems of equations, one can judge the number of intersection points. The difficulty level of this problem is moderate, primarily testing knowledge such as the properties of hyperbolas, the application of the point difference method, and the positional relationship between lines and hyperbolas. To solve this problem, one should first use the known conditions of the hyperbola equation based on the given information, apply the point difference method, set the coordinates of two points into the hyperbola equation, then subtract the two equations to obtain an expression related to the midpoint of the chord and the slope of the line. Finally, analyze each option based on the characteristics of the hyperbolas asymptotes. AB

3.3.2 Solution analysis (standard answer)

First, set up the midpoint, $A(x_1,y_1),B(x_2,y_2)$ AB $M(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

Then there is; $k_{AB}=\frac{y_1-y_2}{x_1-x_2},\ k=\frac{\frac{y_1+y_2}{2}}{\frac{x_1+x_2}{2}}=\frac{y_1+y_2}{x_1+x_2}$

Two: because on the hyperbola, then subtracting the two equations gives, $A,B\begin{cases}x_1^2-\frac{y_1^2}{9}=1\\x_2^2-\frac{y_2^2}{9}=1\end{cases}(x_1^2-x_2^2)-\frac{y_1^2-y_2^2}{9}=0$

so.$k_{AB}\cdot k=\frac{y_1^2-y_2^2}{x_1^2-x_2^2}=9$

1) Option A is true

(a) If it is easy to know;$k=1,k_{AB}=9AB\colon y=9x-8$

(b) Solving the simultaneous equation and eliminating y gives us this; $\begin{cases}y=9x-8\\x^2-\frac{y^2}{9}=1\end{cases}72x^2-2\times 72x+73=0\Delta=(-2\times 72)^2-4\times 72\times 73=-288<0$

(c) Therefore, the straight line AB has no intersection with the hyperbola, so A is wrong;

2) Option B is true

(a) If it is easy to know, then; $k=-2,k_{AB}=-\frac{9}{2}AB\colon y=-\frac{9}{2}x-\frac{5}{2}$

(b) Solve the simultaneous equation, eliminate y, at this time; $\begin{cases}y=-\frac{9}{2}x-\frac{5}{2}\\x^2-\frac{y^2}{9}=1\end{cases}45x^2+2\times 45x+61=0\Delta=(2\times 45)^2-4\times 45\times 61=-4\times 45\times 16<0$

(c) Therefore, the straight line AB has no intersection with the hyperbola, so B is wrong;

3) Option C is true

(a) If it is easy to know;$k=3,k_{AB}=3AB\colon y=3x$

(b) Combined with the hyperbola equation, then it is the asymptote of the hyperbola;$a=1,b=3AB\colon y=3x$

(c) Therefore, the straight line AB has no intersection with the hyperbola, so C is wrong;

4) Option D is true

(a) $k=4,k_{AB}=\frac{9}{4}$ Then; $AB\colon y=\frac{9}{4}x-\frac{7}{4}$

(b) Solving the system of equations and eliminating y, we get, $\begin{cases}y=\frac{9}{4}x-\frac{7}{4}\\x^2-\frac{y^2}{9}=1\end{cases}63x^2+126x-193=0\Delta=126^2+4\times 63\times 193>0$

(c) Therefore, the straight line AB intersects the hyperbola at two points, so D is correct.

3.3.3 Evaluation and reflection

This question comprehensively and deeply tests knowledge, covering core concepts and examining deeper thinking methods. It covers relevant knowledge about hyperbolas, including their standard equations, properties, and the positional relationship between lines and hyperbolas. By integrating these concepts, it assesses students overall grasp of the chapter on hyperbolas. Additionally, it focuses on the point-difference method, an important mathematical technique that demonstrates the requirement for logical reasoning in mathematics. The difficulty level of the questions is well-set. Through this problem, students need to skillfully use the point-difference method to establish the relationship between the slope of a line and the coordinates of the midpoint of a chord, thereby determining whether the given point could be the midpoint of a chord. This places high demands on students mathematical thinking abilities and effectively distinguishes learning levels among different students.

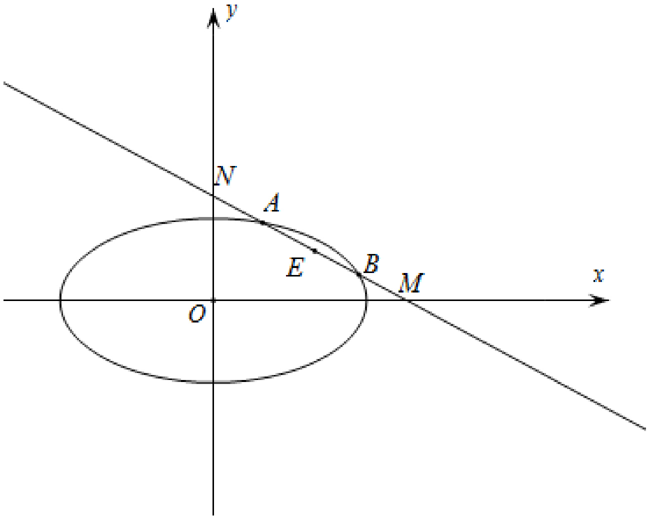
3.4 Intersection Chord Problem of Conic Sections

Example 4: (2022 New College Entrance Examination Paper II, Question 16)

It is known that the line l intersects the ellipse at A and B in the first quadrant, and intersects the x-axis and y-axis at M and N respectively, and then the equation of l is ___________.$\frac{x^2}{6}+\frac{y^2}{3}=1|MA|=|NB|,|MN|=2\sqrt{3}$

3.4.1 Analyze the questions

This problem is based on the knowledge of finding the equation or slope of an ellipse chord through its midpoint. It is challenging due to the complexity brought by mathematical operations and logical reasoning. The main focus is on testing the equations and properties of lines and ellipses, the concept and operations of vectors, the midpoint coordinate formula and equation thinking, as well as the ability to solve problems through computation. This problem can be solved by setting up equations based on the given informationAB Let the midpoint be,, and then use the point difference method to get. Let the straight line be,, and find the coordinates of and then find and according to the result, we can solve the problem. $EA(x_1, y_1)$ $B(x_2, y_2)$ $k_{OE} \cdot k_{AB} = -\frac{1}{2}$ $AB: y = kx + m$ $k < 0 m > 0 M$ $N |MN| k m$



3.4.2 Solution analysis (standard answer)

1) The midpoint of the first order, because, so; EAB $|MA| = |NB|$ $|ME| = |NE|$

2) Let, then,; $A(x_1, y_1) B(x_2, y_2)$ $\frac{{x_1}^2}{6} + \frac{{y_1}^2}{3} = 1$ $\frac{{x_2}^2}{6} + \frac{{y_2}^2}{3} = 1$

3) So, that is, so, that is, let the line be,,; $\frac{{x_1}^2}{6} - \frac{{x_2}^2}{6} + \frac{{y_1}^2}{3} - \frac{{y_2}^2}{3} = 0$ $\frac{(x_1 - x_2)(x_1 + x_2)}{6} + \frac{(y_1 + y_2)(y_1 - y_2)}{3} = 0$ $\frac{(y_1 + y_2)(y_1 - y_2)}{(x_1 - x_2)(x_1 + x_2)} = -\frac{1}{2}$ $k_{OE} \cdot k_{AB} = -\frac{1}{2}$ $AB: y = kx + m$ $k < 0 m > 0$

4) Let it be, let it be, that, therefore, that, so that, or (discard); $x = 0 y = m y = 0 x = -\frac{m}{k} M\left(-\frac{m}{k}, 0\right) N(0, m) E\left(-\frac{m}{2k}, \frac{m}{2}\right) k \times \frac{\frac{m}{2}}{-\frac{m}{2k}} = -\frac{1}{2} k = -\frac{\sqrt{2}}{2} k = \frac{\sqrt{2}}{2}$

5) And, that is, the solution or (discard), so the line, that is. $|MN| = 2\sqrt{3}$ $|MN| = \sqrt{m^2 + \left(\sqrt{2}m\right)^2} = 2\sqrt{3} m = 2 m = -2 AB: y = -\frac{\sqrt{2}}{2} x + 2 x + \sqrt{2} y - 2\sqrt{2} = 0$

3.4.3 Evaluation and reflection

This question is highly comprehensive and has a high degree of differentiation, integrating multiple key concepts such as linear equations, ellipse equations, vector knowledge, and the midpoint coordinate formula. It emphasizes the assessment of thinking skills and computational abilities, closely aligning with curriculum standards. The problem-solving process involves complex operations like elimination, simplification, and solving quadratic equations after setting up systems of equations, which places high demands on students computational and problem-solving skills. Through this question, it effectively tests students computational techniques, accuracy, and patience, helping to enhance their mathematical operational literacy and meet the requirements for talent selection in college entrance exams. However, the context of the problem is relatively simple and lacks innovation. The main focus is on the intersection of a line and an ellipse in the first quadrant, making the scenario somewhat monotonous and lacking close connections to real life or other subjects; the question type and examination methods are relatively traditional, lacking novel angles or innovative problem-solving approaches.

3.5 Conic Curve Fixed Point Problem

Example 5: (2022 National A paper (Liberal Arts) No.21)

Let F be the focus of the parabola, and let a line through F intersect C at M and N. When MD is perpendicular to the x-axis,. $C: y^2 = 2px (p > 0)$ $D(p, 0)$ $|MF| = 3$

(1) Find the equation of C;

(2) Let the other intersection points of the line with C be A and B respectively, and let the inclination angles of the lines be respectively. When the maximum value is obtained, find the equation of the line AB.MD, ND MN, AB $\alpha, \beta \alpha - \beta$

3.5.1 Analyze the questions

This problem revolves around parabolas, comprehensively examining the equation of parabolas, the positional relationship between lines and parabolas, as well as the application of trigonometric functions. It has rich background and connotations, with high difficulty, making it the centerpiece of that years exam. This question is divided into two sub-questions. The first sub-question tests the definition and trajectory equation of parabolas; according to the definition, direct solution can be obtained. The second sub-question focuses on students mastery of trigonometric function applications and maximum/minimum problems, as well as their logical reasoning and comprehensive analytical skills. Based on the coordinates of the point and the line provided in the question, the solutions can be derived using Vietas formula and the slope formula. Then, by applying the tangent formula for difference angles and basic inequalities, the final solution is obtained by setting up the line and combining it with Vietas formula. $|MF| = p + \frac{p}{2} MN: x = my + 1$ $k_{MN} = 2k_{AB}$ $k_{AB} = \frac{\sqrt{2}}{2} AB: x = \sqrt{2} y + n$

3.5.2 Solution analysis (standard answer)

1) The first question

(a) According to the parabolas focus, when it is perpendicular to the x-axis, the horizontal coordinate of point M is p, $x = -\frac{p}{2} MD$

(b) Therefore, the equation of parabola C is; $|MF| = p + \frac{p}{2} = 3$ $p = 2$ $y^2 = 4x$

2) The first question

(a) Set, straight line; $M\left(\frac{y_1^2}{4},y_1\right)$, $N\left(\frac{y_2^2}{4},y_2\right)$, $A\left(\frac{y_3^2}{4},y_3\right)$, $B\left(\frac{y_4^2}{4},y_4\right)$ $MN: x=my+1$

(b) Thus, by the slope formula, we have; $\begin{cases} x=my+1 \\ y^2=4x \end{cases}$ $y^2-4my-4=0\Delta>0, y_1y_2=-4$ $k_{MN}=\frac{y_1-y_2}{\frac{y_1^2}{4}-\frac{y_2^2}{4}}=\frac{4}{y_1+y_2}$ $k_{AB}=\frac{y_3-y_4}{\frac{y_3^2}{4}-\frac{y_4^2}{4}}=\frac{4}{y_3+y_4}$

(c) There is also a straight line, substitute into the parabolic equation can get, so, similarly, we can get, so; $MD: x=\frac{x_1-2}{y_1}\cdot y+2$ $y^2-\frac{4(x_1-2)}{y_1}\cdot y-8=0$ $\Delta>0, y_1y_3=-8$ $y_3=2y_2$ $y_4=2y_1$ $k_{AB}=\frac{4}{y_3+y_4}=\frac{4}{2(y_1+y_2)}=\frac{k_{MN}}{2}$

(d) Because the inclination angles of MN and AB are respectively, α,β $k_{AB}=\tan\beta=\frac{k_{MN}}{2}=\frac{\tan\alpha}{2}$

(e) If we want to maximize, lets set, then, $\alpha-\beta\beta\in\left(0,\frac{\pi}{2}\right)$ $k_{MN}=2k_{AB}=2k>0$ $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\frac{k}{1+2k^2}=\frac{1}{\frac{1}{k}+2k}\leq\frac{1}{2\sqrt{\frac{1}{k}\cdot 2k}}=\frac{\sqrt{2}}{4}$

(f) The equality holds if and only if the time is instant, so when it is maximum, let the straight line, substitute into the parabolic equation, we get, $\frac{1}{k}=2kk=\frac{\sqrt{2}}{2}$ $\alpha-\beta$ $k_{AB}=\frac{\sqrt{2}}{2}$ $AB: x=\sqrt{2}y+n$ $y^2-4\sqrt{2}y-4n=0$

(g) $\Delta>0, y_3y_4=-4n=4y_1y_2=-16$So, so straight line. $n=4AB: x=\sqrt{2}y+4$

3.5.3 Evaluation and reflection

The key to solving this problem lies in simplifying the slope using the parabolic equation and deriving the relationship between coordinates through Vietas formula. From a knowledge perspective, this question comprehensively tests core concepts of parabolas, including their definition, equation, focus, and other fundamental concepts, as well as the positional relationship between lines and parabolas. It also involves the application of trigonometric functions, the inclination angle of a line, and slope, integrating multiple important knowledge points organically. This deeply examines students mastery of knowledge and their ability to apply it comprehensively. In terms of assessing thinking skills, deriving the parabolic equation from given conditions requires strong logical reasoning; solving for intersection points by combining linear and parabolic equations tests algebraic operations and equation-solving ideas; finding the maximum value of the difference in inclination angles using trigonometric functions involves transformation and reduction techniques, as well as the ability to analyze function extremum values. Such diverse assessments of thinking help cultivate students mathematical thinking qualities. Additionally, the problem emphasizes various mathematical methods, ingeniously combining analytic geometry with trigonometric functions, showcasing the intrinsic connections between different areas of mathematics. However, the computational load is substantial; during the solution process, especially in solving systems of equations, applying Vietas formula, and finding extremum values, there are many algebraic operations and simplifications involved.

4. Discussion and Recommendations

4.1 Consolidate the Foundation and Return to the Textbook

Necessary knowledge reserves are the primary foundation for problem-solving. Only by being familiar with and mastering basic knowledge can one skillfully apply knowledge points to solve problems. Many scholars have conducted surveys analyzing and categorizing the reasons for most students "problem-solving errors." Most of these studies point out that the main issues are errors in knowledge and oversights, highlighting phenomena such as confusion between definitions, incorrect application of properties, and inadequate understanding or weak foundations in basic knowledge; there are also manifestations of insufficient computational skills, such as errors in complex expression operations and improper selection of calculation methods; some students lack sufficient thinking abilities and overlook implicit conditions. At the same time, textbooks are closely linked to curriculum standards and serve as their concrete implementation. They are the primary basis for teachers instruction and an important source of knowledge acquisition for students, covering foundational knowledge, basic concepts, fundamental principles, typical examples, and exercises. The aim is to help students gradually build a systematic understanding and recognition of the subject. In principle, college entrance examination questions will not exceed the content summarized in the textbook, so the importance of textbooks is particularly prominent when studying conic sections. We should place greater emphasis on the foundational content of textbooks, using them as a starting point to explore the key points of the new college entrance examination, avoiding blind learning by students.

4.2 Delve into the Essence and Explore by Category

The title is one of the effective ways to examine knowledge and skills. The essence of solving problems should lie in deeply understanding and recognizing the nature of the question, conducting categorized exploration and research. By analyzing the conic section questions in the new national college entrance examination, we can clearly understand that all conic section problems, after being classified and analyzed, can also be traced with a clear "thread." For conic section problem types, teachers and students should summarize and categorize the knowledge involved; focusing on "changing the form but not the substance," enhancing students innovative abilities and independent research capabilities, ultimately promoting the progress and development of modern society. From the teachers perspective, the definition of conic sections is a vivid manifestation of their essential characteristics. Only by clarifying the core definition and starting from it can various properties of conic sections be derived, achieving a deep understanding of the definition and essence of conic sections. In daily teaching, the essence and significance of "learning" must be clarified, and teaching should be conducted from a higher perspective. From the students perspective, problems involving "conic sections" are often one or a type of problem —— they are more about knowledge, requiring students to apply what they learn to solve real-life and work-related issues, thereby promoting comprehensive development of their abilities.

4.3 Clarify Direction and Master Skills

After summarizing and analyzing knowledge about conic sections and corresponding test questions, there should be a general "outline" and direction. Next, for different types of questions, develop corresponding problem-solving steps and techniques based on methods such as definition, setting up without solving, parametric equation method, and point difference method, to face the complex "new college entrance examination." Clear problem-solving steps from students indirectly reflect their problem-solving direction and thinking, directly indicating their familiarity with basic knowledge, which is also a necessary requirement for standardized answers. Regarding the major types of conic sections, the problem-solving ideas are relatively clear, and techniques and methods are quite evident, showing a certain degree of "pattern." Therefore, teachers should first organize teaching, master the techniques, and then discuss them with students together. Before students grasp these problem-solving techniques, they should have a clear direction and "path"; more importantly, they should build their own "high-rise building" on the foundation of mastering basic knowledge through flexible application of problem-solving methods. Teachers should carefully study real college entrance exam questions and consciously guide students in daily teaching to enhance their ability to independently analyze and think about problems and solve them independently.

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