DOI: 10.53469/jerp.2024.06(11).17

A Teaching Design on "Classical Probability Model"

Xiaoyan Dong

Beijing Institute of Petrochemical Technology, Beijing, China
dongxiaoyan@bipt.edu.cn

Abstract: ***This article presents a teaching design scheme for classical probability models. It conducts targeted analysis and training on several aspects where students are prone to make mistakes when learning classical probability models. It is hoped that through comparative analysis of different solutions to typical examples, students can greatly improve in terms of calculation accuracy and simplicity. The mistakes made by students mainly include misjudging the equal possibility of sample points, inaccurate reduction of the sample space, and being unable to choose the simplest method when choosing permutation or combination for calculation.***

Keywords: Classical probability model, Sample space, Permutation, Combination.

1. Introduction

Classical probability model is the basic content of probability theory and plays an important role in subsequent probability calculation. Classical probability model requires that randomized trials meet two conditions: the sample space contains only a finite number of sample points, and each sample point has the same probability of occurrence. In general teaching of classical probability model, this phenomenon often occurs: when the teacher explains the examples, the students can understand them quickly, but when the students do the problems themselves, there are various mistakes. A careful analysis of the reasons why students make mistakes is that on the one hand, they do not know the same possibility whether the sample points are satisfied, and on the other hand, they have misunderstanding about when the reduced sample space can be used. In addition, they are unclear whether to use permutation or combination in the specific calculation of some examples. Therefore, the teacher made the following teaching design. In view of the above problems of students, several typical questions of classical probability model are selected, and different solutions are adopted respectively to increase the process of comparative analysis and step by step to help students strengthen their understanding of their own mistakes. Let students establish a good habit of consciously judging the same possibility of sample points when doing problems, select a complete sample space when it is not clear whether the sample points have sufficient symmetry, and make appropriate selection of permutation or combination to complete the calculation as concisely and accurately as possible.

2. Classical Probability Model and Formulas [1]

Let random experiment E meet the following two conditions:

The sample space contains n (n is a finite number) sample points.

Each sample point is equally likely to happen.

Then random experiment E is the classical probability model.

If the random event A contains k sample points, then $P(A)=\frac{k}{n}$.

3. Comparative Solution of Several Typical Examples

3.1 An Example of Tossing a Coin

Example 1. Toss a uniform coin at random three times, let random events A_1 means exactly one head, random events A_2 means at least one head, find $P(A_1)$ and $P(A_2)$.

Solution 1: Since both the random events A_1 and A_2 involve the number of occurrences of heads, students are likely to think of taking the test result of the random experiment, that is, the sample point, as the number of occurrences of heads in three flips. Then the sample space is S={0,1,2,3} .Then $P(A_1)=\frac{1}{4}$, $P(A_2)=\frac{3}{4}$.

Solution 2: Let's call it H for heads, T for tails. Taking the cases of both sides in three flips as sample points, then the sample space is S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}. Then $P(A_1)=\frac{3}{8}$, $P(A_2)=\frac{7}{8}$.

When students compare the above two solutions, they find that the sample spaces determined by choosing different observation angles are also different. Some of them make the sample points have equal possibility, while others make the sample points not have equal possibility. After comparison, it can be clearly found that the sample points 0 and 1 in Solution 1 are not equally likely to occur. Then, in Solution 1, observing the experimental result from the perspective of "observing the number of heads appearing in three flips when a coin is tossed three times", at this time the random experiment is not a classical probability model, so Solution 1 is wrong. While in Solution 2, "tossing a coin three times and observing the situation of heads and tails appearing", since the heads and tails of a uniform coin have symmetry, the eight sample points are completely equally likely, so Solution 2 is correct. The comparative analysis of this question helps students realize that instead of just starting from the question's wording, the sample space should be constructed based on equal possibility.

3.2 An Example of Taking Balls Where the Numbers of Balls of Different Colors are Different

Example 2: There are 6 balls of the same size and texture in the box, of which 4 are white balls and 2 are red balls. Now take the balls randomly twice, one at any time, and consider the way of taking the balls back to sampling. Set event A: "Both balls are white balls", and find P(A) .

Solution 1: The question asks the test results of "both balls are white". So the sample space naturally considers the color distribution of the two balls. That is, the sample space is constructed as S={white white, white red, red white, red red}. Then $\mathrm{P(A)} = \frac{1}{4}$.

Solution 2: Number each ball in the box so that the 4 white balls are "1, 2, 3, 4" and the 2 red balls are "5, 6". The number of the two balls taken out in sampling with replacement, that is, the binary ordered number pair, is taken as the sample point. So the sample space is constructed as S ={(1,1),(1,2),(1,3)...(1,6),(2,1),(2,2),...(2,6),...(5,6),(6,6)}. Then P(A)= $\frac{4\times4}{6\times6} = \frac{4}{9}$.

When comparing the above two solutions, it is easy for students to point out that since the number of white balls in the box is not equal to the number of red balls, "white white" and "red red" in solution 1 are obviously not equally likely to happen, so solution 1 is wrong. Solution 2 takes every ball into account equally, making every sample point in the sample space equally likely, so solution 2 is correct. The students concluded that the unequal number of white and red balls in the box was the reason for the error of solution 1. In order to make students realize that this attribution is not comprehensive, the teacher gives example 3 based on Example 2.

3.3 An Example of Taking Balls Where the Number of Balls of Different Colors is the Same

Example 3: There are 6 balls of the same size and texture in the box, of which 3 are white balls and 3 are red balls. Now take the balls randomly twice, one at random each time. Consider the two ways of taking the balls. That is sampling with replacement and sampling without replacement. Let event A: "Both balls taken are white balls", find the probability of event A.

3.3.1 At the case of sampling with replacement:

Solution 1:

Let the sample space S_1 = {{white white, white red, red white, red red}}, then $\mathrm{P(A)} = \frac{1}{4}$.

Solution 2:

Number each ball in the box, with the 3 white balls numbered "1, 2, 3" and the 3 red balls numbered "4, 5, 6". Let the sample space S_2 = {(1,1),(1,2),...(1,6),(2,1),(2,2),...(2,6),...,(6,6)}. Then P(A)= $\frac{3\times3}{6\times6} = \frac{1}{4}$.

In this example, since the number of white balls and red balls is the same, and the sampling method is sampling with replacement, so the sample space S_2 is equal proportional reduction of the sample space S_1. So the above two solutions are both correct.

3.3.2 At the case of sampling without replacement:

Solution 1:

Let the sample space S_1={{white white, white red, red white, red red}},then $\mathrm{P(A)} = \frac{1}{4}$.

Solution 2:

Number each ball in the box, with the 3 white balls numbered "1, 2, 3" and the 3 red balls numbered "4, 5, 6". Since now is sampling without replacement, the same ball number will not occur twice. Let the sample space S_2 = {(1,2),(1,3)...(1,6),(2,1),(2,3),...(2,6),...(5,6}. Then P(A) = $\frac{3\times2}{6\times5} = \frac{1}{5}$.

When sampling without replacement, solution 1 is wrong and solution 2 is correct.

At the case of sampling with replacement, since there are equal numbers of red and white balls, the probability of the four sample points in solution 1 is equal, so it is still correct to reduce the sample space of solution 2 to that of solution 1. However, once sampling is that without replacement, it is no longer equally possible to take the white ball or take the red ball the second time after the first retrieval, and the four sample points of solution 1 no longer have the same possibility, so solution 1 is wrong. At this time, the complete sample space S_2 cannot be reduced to S_1 for ease of calculation. Through example 3, students realize that when constructing the sample space of the classical probability model, it is not enough to only look at whether the number of red balls and white balls is equal. The sampling method of the random experiment also needs to be considered. At the same time, students also realize that in such ball-taking problems, numbering each ball so as to view them independently and differently means that the symmetry is completely satisfied when considering the complete sample space. This is the fundamental method to ensure that the sample points occur equally likely. However, the method of reducing this complete sample space based on the property that the number of balls of different colors is equal has limitations. In cases such as sampling without replacement, due to the loss of symmetry, this seemingly simpler approach can lead to errors. Therefore, if there is no absolute certainty, in ball-taking problems, numbering each ball separately and considering them all, that is, choosing the complete sample space, is a safe and accurate method.

3.4 An Example of Taking Products with Genuine and Defective Items

Example 4: There are 7 products of the same model, of which 3 are defective and 4 are genuine. Take 4 products from any of these products and find the probability of the event "A: There are exactly 2 defective products out of 4 products".

Solution 1: Number each product, assuming that "1, 2, 3" is defective, and "4, 5, 6, 7" is genuine. The random experiment in this case is to take 4 of the 7 products and observe the distribution of the numbers taken out, and the experimental result is 4 different numbers between 1 and 7.Since the event

A required only considers the number of defective products in the four products, it can be considered that the test result has nothing to do with the order, then each result taken out is an unordered array composed of four different numbers, so the combination number is used to calculate. Then the total number of basic events in the sample space is C_7^4. The number of basic events contains in event A is $C_3^2 \cdot C_4^2$. So $P(A) = \frac{C_3^2 . C_4^2 .}{C_7^4} = \frac{18}{35}$.

Solution 2: Number each product, assuming that "1, 2, 3" is defective, and "4, 5, 6, 7" is genuine. In this case, the random experiment is regarded as taking 4 products from 7 products. The ordered distribution of the serial numbers of the 4 products taken out is considered. Then the experimental results are related to the order and need to be calculated by using permutation numbers. Then the total number of basic events in the sample space is A_7^4 .The number of basic events contains in event A is$A_3^2 \cdot A_4^2 + A_4^2 \cdot A_3^2 + A_3^1 \cdot A_4^2 \cdot A_2^1 + A_4^1 \cdot A_3^2 \cdot A_3^1 + A_3^1 \cdot A_4^1 \cdot A_2^1 \cdot A_3^1 + A_4^1 \cdot A_3^1 \cdot A_3^1 \cdot A_2^1 = 432.$ The basic event contained in event A is very tedious and complicated due to the various arrangement of serial numbers of the 2 defective items and 2 genuine items taken out, Includes the following six kinds of situations. "the first two defective items and then two genuine items". "First two genuine items and then two defective items". "First one defective item then two genuine items and then one defective item". "First one genuine item then two defective items and then one genuine item". "First a defective item and a genuine item and then also a defective item and a genuine item". "First a genuine item and a defective item and then also a genuine item and a defective item". According to the probability calculation formula of classical probability model, $P(A) = \frac{432}{A_7^4} = \frac{18}{35}$.

The two solutions to Example 4 are both correct. However, it is obvious that Solution 1 is simple, direct, and less prone to errors. In this question, analyzing the results of a random experiment can be regarded as independent of the serial number of the product taken. In this case, using combination calculation is more convenient. It is possible to consider the order using permutation, but it will be very cumbersome and unnecessary. Therefore, when encountering a situation where both combination and permutation can be used for calculation, combination must be chosen first. When the order of experimental results need not be considered, do not consider the order. Calculating with combination will greatly simplify the calculation.

4. Conclusion

Through the four carefully designed examples above, students have gained a more comprehensive understanding of the sample space and the choice of calculation methods, and have a clearer understanding of the areas prone to errors when calculating probabilities in classical probability models. By doing more exercises, choosing an appropriate perspective to construct the sample space during practice and consciously conducting training on the equal possibility of basic events, it is believed that students' learning of classical probability models will be significantly improved.

References

[1] Sheng Zhou, Xie Shiqian, Pan Chengyi, Probability theory and mathematical statistics (Fourth Edition), Higher Education Press, 2016.